Laser-Powered UAV Trajectory and Charging Optimization for Sustainable Data-Gathering in the Internet of Things

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Abstract—This work examines the trajectory design and energy charging strategy of a data-gathering unmanned aerial vehicle (UAV). The UAV utilizes laser charging from high-altitude platforms (HAPs) to replenish its battery, enabling sustained travel across multiple data-gathering points. The trajectory is determined by a sequence of hovering positions at which the UAV stays to perform both data collection and energy charging. The UAV's hovering positions affect both the sensors' transmission rates and the laser-charging efficiency. To minimize the total task completion time, it is necessary to choose hovering positions that consider both data upload and energy charging times. In this work, we first propose the Minimum Completion Time Trajectory and Charging Optimization (MinTime-TCO) algorithm, where the hovering positions and charging energies are optimized in turn using a block coordinate descent approach. Given the UAV's hovering positions, we propose the Minimum Charge Rate Search (MCRS) algorithm to optimize the charging energies at these positions. We show that MCRS is optimal in terms of minimizing the total task completion time. Then, given the charging energies, we propose the Hovering Position Optimization (HPO) algorithm, employing successive convex approximation to address the non-convexity of the optimization problem. We also propose a low-complexity alternative based on dynamic programming to further reduce computational complexity. Simulation results demonstrate the effectiveness of the proposed algorithms against several baseline strategies.

Index Terms—UAV communication, Internet of Things, laser charging, successive convex approximation, dynamic programming.

I. INTRODUCTION

U NMANNED aerial vehicles (UAVs) serve as an efficient means of data-gathering from the Internet of Things (IoT) or wireless sensor networks (WSNs) due to their high mobility and ease of deployment [1], [2], [3]. The mobility allows UAVs to fly close to sensors for data collection, significantly reducing the transmit power consumption of sensors, and thus

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extending their lifetimes. However, the UAVs' limited battery capacity and substantial propulsion energy consumption limit their flight range and coverage. Several works, e.g., [4], [5], assumed the availability of fixed charging stations on the ground and optimized their data-gathering operations accordingly. However, IoT sensors or devices may be deployed in areas with limited infrastructure, such as forests, battlefields, and disaster sites. In these cases, deploying and maintaining a fixed charging facility on the ground would be costly and could impede data-gathering efficiency by necessitating frequent UAV trips to the charging stations for battery replenishment. Hence, integrating efficient wireless charging technology [6], [7] is essential to offer increased flexibility in the trajectory design and sustain the data-gathering operation over a large area.

Many works in the literature, e.g., [8], [9], [10], [11], explored the use of wireless power transfer (WPT) within the radio frequency (RF) band for the remote charging of UAVs. The UAVs' transmit resources and flight trajectories were often jointly optimized to reduce the total mission cost, such as task completion time or total energy consumption. However, RF wireless charging is limited to short distances due to the rapid dispersal of electromagnetic waves over the air and, thus, imposes strict constraints on the trajectory design of the UAVs. To increase the charging range, recent literature adopted laser charging as a promising alternative to RF charging, using a more concentrated beam for energy transfer [12], [13]. The demonstration by PowerLight Technologies [14] shows that laser charging can support UAV energy over hundreds of meters, providing more flexibility for the UAV trajectory design. Moreover, [13] showed that laser charging is particularly effective for data-gathering applications, where UAVs must cover extensive areas compared to RF wireless or tethered charging solutions. Most works, e.g., [15], [16], [17], [18], [19], [20], [21], [22], assume that the laser beam director that emits the laser beam to the UAV is located on the ground and may be susceptible to blockage by buildings, trees, or other obstacles. Consequently, several other works, such as [23], [24], explored the feasibility of mounting the laser beam director on high altitude platforms (HAPs), allowing for more efficient energy harvesting through technologies like large solar panels or wind turbines. While practical challenges remain in deploying laser charging stations on HAPs with the current technology, several organizations (such as PowerLight Technologies [14] and the Japan Aerospace Exploration Agency (JAXA) [25])

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continue to advance the technology and will eventually enable more compact and efficient charging systems for HAPs.

This work examines the trajectory design and energy charging strategy of a laser-powered UAV for data-gathering in IoT or WSNs. Laser charging is facilitated by HAPs deployed at fixed positions above the sensor field to support data-gathering across all sensors. The trajectory design involves optimizing a set of hovering positions where the UAV stays to collect sensor data while replenishing its battery through laser charging. The selected hovering positions impact both the transmission rates of sensors and the efficiency of laser charging by the HAPs. In this work, we aim to minimize the overall task completion time by optimizing the hovering positions of the UAV and its charging energies at these positions, taking into account both data upload and energy charging times at different points. The main contributions of this paper are summarized as follows:

- We propose the Minimum Completion Time Trajectory and Charging Optimization (MinTime-TCO) algorithm where the UAVs' hovering positions and charging energies are optimized in turn using a block coordinate descent (BCD) approach to minimize the time required to gather data from all sensors.
- Given the UAV's hovering positions, we propose the Minimum Charge Rate Search (MCRS) algorithm to determine the optimal charging energies at various points. We prove that the MCRS algorithm is optimal in terms of minimizing the total task completion time.
- Then, given the charging energies, we propose the Hovering Position Optimization (HPO) algorithm, employing a successive convex approximation (SCA) method to address the non-convexity of the objective and constraints. This is referred to as the HPO-SCA algorithm.
- To reduce the computational complexity, we also propose a low-complexity HPO algorithm based on the dynamic programming (DP) principle. This approach, referred to as HPO-DP, yields complexity that scales linearly with the number of data-gathering points.
- Finally, we provide simulation results to demonstrate the effectiveness of the proposed algorithms against several baseline approaches.

The remainder of this paper is organized as follows. Section II reviews existing works on UAV communications that utilize RF or laser charging to support the UAV's energy usage. Section III describes the system model and problem formulation. Section IV proposes the MinTime-TCO algorithm, which alternates between the MCRS and HPO-SCA algorithms in turn to find the optimal hovering positions and charging energies. Section V proposes the HPO-DP algorithm as a low-complexity alternative to the HPO-SCA algorithm. The optimality of the MCRS algorithm is then proved in Section VI. Finally, simulation results are provided in Section VII and the paper is concluded in Section VIII.

II. RELATED WORKS

This section reviews several works that utilize far-field WPT technology, such as RF wireless charging or laser charging, to

support the UAV's energy usage. While these works present interesting solutions to sustain longer UAV operations, the inefficiency of RF wireless charging over long distances and the susceptibility to blockage experienced by ground laser chargers highlight the need to deploy laser chargers in the sky, which is the focus of our work.

Specifically, several works in the literature, e.g., [8], [9], [10], [11], considered the use of RF wireless charging to sustain the operations of UAV-enabled base stations or data-gathering nodes. Reference [8] employed a wireless RF charging station on the ground to power a UAV-enabled aerial base station. The worst-case throughput of the ground users was maximized by jointly optimizing resource allocation and UAV placement. The work in [9] extended the problem to systems with multiple UAV-enabled aerial base stations. The downlink sum rate of the aerial base station was maximized by jointly optimizing user association, resource allocation, and base station placement. Moreover, [10] introduced a non-disruptive wireless rechargeable UAV network model that allows UAVs to be charged during flight by wireless chargers, eliminating the need to hover at fixed positions or return to dedicated charging platforms. To minimize the energy waste of wireless chargers, a heuristic exhaustive candidate solution was proposed where a charger tends to simultaneously charge as many UAVs as possible. Reference [11] considered using two types of UAVs: charging UAVs and mission UAVs, where the former was used to charge the latter without interrupting the mission. The total time required for mission UAVs to complete their tasks was minimized by scheduling charging times and planning travel paths for charging UAVs using deep reinforcement learning. In the above works, RF wireless charging was adopted to facilitate far-field charging of UAVs without returning to dedicated charging platforms and, thus, provide more flexibility for the UAVs' trajectory designs. However, the rapid dispersal of RF waves can significantly reduce energy transfer efficiency, especially in far-field charging scenarios.

To improve the efficiency of wireless charging, recent works, e.g., [15], [16], [17], [18], [19], [20], [21], [22], have adopted ground laser chargers to provide the long-distance energy charging required by UAVs. In particular, [15] considered the communication between a laser-powered UAV and a ground station and aimed to maximize the downlink throughput between the two terminals by optimizing the UAV's trajectory at a fixed altitude. [16] examined the resource allocation for a UAV relay that assists the transmission between a ground terminal and many IoT devices. The ground terminal provides both a free-space optical link and a high-power laser beam to power the UAV. The laser charging time, data upload time, and relay power allocation were jointly determined to maximize the number of IoT devices that can be served. [17] examined a similar UAV relay problem and proposed to maximize the energy efficiency of both information transmission and laser charging by jointly determining the transmit and charging powers and the UAV trajectory. Ref. [18] considered an ultra-reliable and low latency communications (URLLC) application with a laser-powered UAV as a relay between a ground controller and a robot on the ground acting as the receiver. The decoding error rate was minimized by jointly optimizing the transmit powers, UAV trajectory, and code length. Moreover, in [19], a laser-powered UAV base station was used to serve multiple ground users through simultaneous wireless information and power transfer. A clustering-based hybrid multiple access technique was proposed, combining both orthogonal and non-orthogonal multiple access schemes. Ref. [20] proposed a deployment strategy for laser-powered UAV base stations and analyzed the resulting coverage using tools from stochastic geometry. The user's connectivity profile was analyzed under different laser charging capabilities and optical turbulence. Ref. [21] considered the use of a UAV relay between a ground base station and multiple ground users and proposed a cost-aware UAV placement strategy to ensure quality communication and energy links for the UAV, ground devices, and users. The UAV was powered by both laser beam directors on the ground and local renewable energy sources. Furthermore, [22] examined the use of UAV-enabled flying energy sources, powered by ground laser chargers, to support a set of mission UAVs through RF wireless charging. The positions of the flying energy sources were determined by a multi-agent deep deterministic policy gradient method, considering both fairness and energy consumption.

While the above works show the efficacy of laser charging in prolonging the lifetime of UAV networks, these works assumed that the laser beam directors are located on the ground, making them susceptible to blockage by buildings, hilly terrain, or smoke. This motivates the deployment of laser chargers in the sky to establish a more direct path between the laser beam director and the energy receiver. Several recent works (e.g., [23], [24]) have thus considered the use of HAPs to serve as the aerial laser charging station. In particular, [23] considered a multi-drone-enabled data collection system that consists of several Low Altitude Platforms (LAPs) collecting data from IoT devices and a HAP that provides energy to the LAPs through laser charging. The total laser charging energy of the HAP was minimized by jointly optimizing the LAPs' trajectory and their laser charging durations, subject to battery constraints. An approximate solution was obtained based on the well-known 2-opt algorithm. However, [23] assumed that laser charging can only be conducted at a fixed position directly below the HAP and, thus, the LAPs (or UAVs) must detour back to this fixed position when charging is required, significantly increasing the mission completion time. [24] considered a HAP-aided multiaccess edge computing system where the HAP delivers energy by laser charging to aerial users and each aerial user then uses the energy to maintain flight operation and execute or offload computation tasks. By considering a terrestrial adversary that distributes false charging locations to aerial users, a Colonel Blotto game framework was adopted to examine the dynamics of the attack-defense interaction between the adversary and the defender (i.e., HAP). This work focused on the privacy awareness and attack prevention, and did not address the data gathering problem considered in our work.

Notice that most works mentioned above consider using UAVs as aerial base stations or relays and aim to maximize the throughput or minimize the energy consumption for



Fig. 1. Illustration of the proposed UAV data-gathering scenario in which a UAV visits J hovering positions to collect data from the J sensors while replenishing its battery through laser charging by the I HAPs.

serving ground users. In contrast, our work explores the use of laser-powered UAVs specifically designed for data gathering in IoT networks. In this case, the trajectory design of the UAV must consider the locations and data sizes of sensors alongside the charging efficiency at various hovering points. Moreover, different from [20], we allow energy storage at the UAV to preserve the energy received at efficient charging points for later use during the data-gathering operation. We also impose a constraint on the UAV's battery capacity, introducing additional challenges in the optimization process. The proposed MCRS algorithm determines the optimal charging policy under these constraints and, thus, is a highlight contribution of this work.

III. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a UAV-enabled data-gathering scenario, as shown in Fig. 1, where a UAV is dispatched to collect data from J sensors on the ground. The sensors' locations are fixed with coordinates given by $\mathbf{s}_j = [s_{j,1}, s_{j,2}, 0]$, for $j = 1, \dots, J$. The UAV collects data from the sensors in turn by visiting the positions $u_1, u_2, ..., u_J$, where $u_j = [u_{j,1}, u_{j,2}, u_{j,3}]$ is the position at which the UAV hovers when collecting data from sensor j. The UAV's height $u_{j,3}$ is restricted within minimum and maximum values H_{\min} and H_{\max} to account for operational regulations. We assume that the visiting order is determined a priori (e.g., by solving the Traveling Salesman Problem) and that the sensors are labeled accordingly. To provide the energy necessary for the UAV to traverse the network, we also deploy IHAPs, each carrying a long-distance laser-charging system, to charge the UAV remotely. The *i*-th HAP is located at coordinates $\mathbf{a}_i = [a_{i,1}, a_{i,2}, a_{i,3}]$ and is assumed to be relatively stationary compared to the speed of the UAV. In this work, we assume that laser charging occurs only at the UAV's hovering positions (i.e., $\mathbf{u}_1, \ldots, \mathbf{u}_J$) to avoid the need to track the UAV's movement and align the laser beams in real-time, which may be challenging in practice.

A. Communication Model

For the communication channel, we adopt the probabilistic path loss model in [21], [26], where the average channel gain between the UAV and sensor j can be expressed as

$$\bar{g}_{j}(\mathbf{u}_{j}) \triangleq \Phi_{j}^{\mathrm{LoS}}(\mathbf{u}_{j}) \frac{\rho_{0}^{\mathrm{LoS}}}{\|\mathbf{u}_{j} - \mathbf{s}_{j}\|^{\alpha^{\mathrm{LoS}}}} + (1 - \Phi_{j}^{\mathrm{LoS}}(\mathbf{u}_{j})) \frac{\rho_{0}^{\mathrm{NLoS}}}{\|\mathbf{u}_{j} - \mathbf{s}_{j}\|^{\alpha^{\mathrm{NLoS}}}}, \qquad (1)$$

where $\Phi_j^{\text{LoS}}(\mathbf{u}_j)$ is the probability of Line-of-Sight (LoS) between sensor j and the UAV, $\rho_0^{\rm LoS}$ and $\rho_0^{\rm NLoS}$ are the reference channel gains at 1 m distance for the LoS and Non-Line-of-Sight (NLoS) channels, respectively, and $\alpha^{\rm LoS}$ and $\alpha^{\rm NLoS}$ are the path loss exponents. Following [21], [26], the probability of LoS is given by

$$\Phi_j^{\text{LoS}}(\mathbf{u}_j) = \frac{1}{1 + C_2 \exp\left\{-C_1 \left[\frac{180}{\pi} \operatorname{arcsin}(\frac{u_{j,3}}{\|\mathbf{u}_j - \mathbf{s}_j\|}) - C_2\right]\right\}},$$
(2)

where $\frac{180}{\pi} \arcsin\left(\frac{u_{j,3}}{\|\mathbf{u}_i - \mathbf{s}_j\|}\right)$ is the elevation angle, C_1 and C_2 are two constant values that depend on the environment. According to [21], [26], (C_1, C_2) can be chosen as (0.429, 4.88), (0.1581, 9.6117) and (0.136, 11.95) for rural, urban, and dense urban environments, respectively.

Following arguments in [27], [28], we assume that the average channel gain is dominated by the LoS term and, thus, can be well approximated by the first term in (1), i.e.,

$$\bar{g}_j(\mathbf{u}_j) \approx \Phi_j^{\text{LoS}}(\mathbf{u}_j) \frac{\rho_0^{\text{LoS}}}{\|\mathbf{u}_j - \mathbf{s}_j\|^{\alpha^{\text{LoS}}}} \triangleq g_j(\mathbf{u}_j).$$
(3)

Consequently, the data rate between the UAV and sensor j can be written as

$$R_j(\mathbf{u}_j) = B \log\left(1 + \frac{P^{\mathrm{tx}}g_j(\mathbf{u}_j)}{BN_0}\right),\tag{4}$$

where B is the bandwidth, P^{tx} is the transmit power, and N_0 is the noise power spectral density. We assume that only one sensor is transmitting in each time slot and, thus, there is no co-channel interference. By letting D_i be the size of the data to be uploaded by sensor j, the upload time of sensor j can be computed as $D_i/R_i(\mathbf{u}_i).$

B. Energy Consumption Model

Here, we consider only the flight propulsion energy consumption of the UAV, since the receiving energy of the communication device is relatively negligible. We assume that the UAV either hovers at a data-gathering point or travels at a constant speed of V m/s from one point to another. The power consumption in these two cases is denoted by P_0 and P_1 , respectively. These values can be calculated, for example, by setting the flight velocities to 0 and V m/s, respectively, under the power consumption model in [29], [30]. The flight time required for the UAV to travel between data gathering points j_1 and j_2 is given by $\frac{\|\mathbf{u}_{j_1}-\mathbf{u}_{j_2}\|}{V}$, and, thus, the corresponding flight energy consumption is $\frac{\|\mathbf{u}_{j_1} - \mathbf{u}_{j_2}\|}{V} P_1$. While it is possible to further optimize the flight velocity over different

segments of the trajectory, for simplicity, we do not take this into account in our work.

C. Charging Model

The energy required for the UAV to traverse multiple datagathering points is provided through laser charging by the I HAPs. We assume that charging can occur only when the UAV is at one of its hovering positions, so as to avoid the need for real-time beam tracking. Hence, we shall often refer to \mathbf{u}_i as the charging (or hovering) position j. Following [31], the charging power received by the UAV from HAP *i* at hovering position *j* can be expressed as

$$p_{i,j}^{\mathrm{ch}}(\mathbf{u}_j) = \frac{\eta P^{\mathrm{L}} \exp(-\gamma \|\mathbf{a}_i - \mathbf{u}_j\|)}{(\zeta + \phi \|\mathbf{a}_i - \mathbf{u}_j\|)^2},$$
(5)

where P^{L} denotes the laser power emitted by the HAPs, η captures the combined efficiency of the energy harvesting circuit, optical transceiver, and receiver area, and γ represents the medium's attenuation coefficient. Additionally, ζ represents the length of the beam, and ϕ represents the angular spread of the beam. By assuming that the UAV is always charged by the closest HAP, the charging power (or charge rate) of the UAV hovering at position j can be expressed as

$$p_j^{\max}(\mathbf{u}_j) = \max_{i \in \{1, \dots, I\}} p_{i,j}^{\text{ch}}(\mathbf{u}_j).$$
(6)

Let $\mathbf{e} \triangleq (e_1, \dots, e_J)$ be the charging policy with e_i being the energy to be harvested by the UAV at position *j*. In this case, the time required to charge the UAV at position j is given by $e_j/p_j^{\max}(\mathbf{u}_j)$. Moreover, since the UAV must remain at hovering position j until the data from sensor j is completely uploaded, the hovering time at position j can be expressed as

$$\max\left(\frac{D_j}{R_j(\mathbf{u}_j)}, \frac{e_j}{p_j^{\max}(\mathbf{u}_j)}\right).$$
(7)

Thus, the energy consumption for the UAV hovering at position $j \operatorname{ismax}(\frac{D_j}{R_j(\mathbf{u}_j)}, \frac{e_j}{p_j^{\max}(\mathbf{u}_j)})P_0$. Suppose that the battery capacity of the UAV is E and the battery is fully charged upon arrival at the initial position \mathbf{u}_1 .

Due to energy causality, the UAV cannot consume energy before it is harvested from the HAPs. Therefore, to sustain travel across multiple data-gathering points, it is necessary to ensure that the total energy harvested by the UAV up to any position j is greater than or equal to the total energy consumed upon arrival at the next position j + 1, i.e.,

$$E_{j+1}^{\text{in}}(\mathbf{e}, \{\mathbf{u}_{j'}\}_{j'=1}^{J}) \triangleq E + \sum_{j'=1}^{j} e_{j'} - \sum_{j'=1}^{j+1} \frac{\|\mathbf{u}_{j'} - \mathbf{u}_{j'-1}\|}{V} P_1$$
$$- \sum_{j'=1}^{j} \max\left(\frac{D_{j'}}{R_{j'}(\mathbf{u}_{j'})}, \frac{e_{j'}}{p_{j'}^{\max}(\mathbf{u}_{j'})}\right) P_0 \ge 0, \tag{8}$$

for $j = 1, \ldots, J$, where $\mathbf{u}_0 = \mathbf{u}_1$ and $\mathbf{u}_{J+1} = \mathbf{u}_J$. This is referred to as the energy causality constraint. The function $E_{i+1}^{\text{in}}(\mathbf{e}, \{\mathbf{u}_{j'}\}_{i'=1}^{J})$ on the left-hand-side (LHS) of (8) is the remaining battery energy upon arrival at position j + 1 under charging policy e and hovering positions $\{\mathbf{u}_{j'}\}_{j'=1}^{J}$.

Moreover, since the UAV cannot be charged beyond its battery capacity E, the remaining energy upon departure from any position j must not be greater than E, i.e.,

$$E_{j}^{\text{out}}(\mathbf{e}, \{\mathbf{u}_{j'}\}_{j'=1}^{J}) \triangleq E + \sum_{j'=1}^{j} e_{j'} - \sum_{j'=1}^{j} \frac{\|\mathbf{u}_{j'} - \mathbf{u}_{j'-1}\|}{V} P_{1}$$
$$- \sum_{j'=1}^{j} \max\left(\frac{D_{j'}}{R_{j'}(\mathbf{u}_{j'})}, \frac{e_{j'}}{p_{j'}^{\max}(\mathbf{u}_{j'})}\right) P_{0} \leq E,$$
(9)

for j = 1, ..., J. This is referred to as the *battery capacity* constraint. The function $E_j^{\text{out}}(\mathbf{e}, \{\mathbf{u}_{j'}\}_{j'=1}^J)$ on the LHS of (9) is the remaining energy upon departure from position j under charging policy \mathbf{e} and hovering positions $\{\mathbf{u}_{j'}\}_{j'=1}^J$. Notice that the remaining energy upon arrival at position j + 1 is equal to that upon departure from position j minus the energy consumed for the flight from \mathbf{u}_j to \mathbf{u}_{j+1} , i.e.,

$$E_{j+1}^{\text{in}}(\mathbf{e}, \{\mathbf{u}_{j'}\}_{j'=1}^{J}) = E_{j}^{\text{out}}(\mathbf{e}, \{\mathbf{u}_{j'}\}_{j'=1}^{J}) - \frac{\|\mathbf{u}_{j+1} - \mathbf{u}_{j}\|}{V}P_{1}.$$
(10)

D. Problem Formulation

The main objective of our work is to determine the optimal charging energies $\{e_j\}_{j=1}^J$ (or e) and hovering positions $\{\mathbf{u}_j\}_{j=1}^J$ that minimize the UAV's total task completion time, subject to energy causality and battery capacity constraints. Note that the task completion time depends not only on the flight time but also on the charging powers and data rates, both of which are influenced by the UAV's hovering positions. The problem can be formulated as

$$\min_{e_j,\mathbf{u}_j,\forall j} \sum_{j=1}^{J} \max\left(\frac{D_j}{R_j(\mathbf{u}_j)}, \frac{e_j}{p_j^{\max}(\mathbf{u}_j)}\right) + \sum_{j=1}^{J-1} \frac{\|\mathbf{u}_j - \mathbf{u}_{j+1}\|}{V}$$
(11a)

subject to
$$E + \sum_{j'=1}^{j} e_{j'} - \sum_{j'=1}^{j+1} \frac{\|\mathbf{u}_{j'} - \mathbf{u}_{j'-1}\|}{V} P_1$$

 $-\sum_{j'=1}^{j} \max\left(\frac{D_{j'}}{R_{j'}(\mathbf{u}_{j'})}, \frac{e_{j'}}{p_{j'}^{\max}(\mathbf{u}_{j'})}\right) P_0 \ge 0, \forall j,$ (11b)

$$E + \sum_{j'=1}^{j} e_{j'} - \sum_{j'=1}^{j} \frac{\|\mathbf{u}_{j'} - \mathbf{u}_{j'-1}\|}{V} P_1$$
$$- \sum_{j'=1}^{j} \max\left(\frac{D_{j'}}{R_{j'}(\mathbf{u}_{j'})}, \frac{e_{j'}}{p_{j'}^{\max}(\mathbf{u}_{j'})}\right) P_0 \le E, \forall j,$$
(11c)

$$H_{\min} \le u_{j,3} \le H_{\max}, \,\forall j. \tag{11d}$$

The problem is non-convex and thus difficult to solve in general. The non-convexity mainly arises due to the terms $e_j/p_j^{\max}(\mathbf{u}_j)$ and $D_j/R_j(\mathbf{u}_j)$ appearing in both the objective and constraints. To address these challenges, we propose an efficient solution based on the BCD approach, where the charging energies and hovering positions are optimized in turn until convergence. The SCA technique is further employed to handle the non-convexity related to the hovering positions. Using the previously defined $E_{j+1}^{\text{in}}(\mathbf{e}, \{\mathbf{u}_{j'}\}_{j'=1}^{J})$ and $E_{j}^{\text{out}}(\mathbf{e}, \{\mathbf{u}_{j'}\}_{j'=1}^{J})$, the constraints in (11b) and (11c) can be expressed as $E_{j+1}^{\text{in}}(\mathbf{e}, \{\mathbf{u}_{j'}\}_{j'=1}^{J}) \ge 0$ and $E_{i}^{\text{out}}(\mathbf{e}, \{\mathbf{u}_{j'}\}_{i'=1}^{J}) \le E$, respectively.

IV. MINIMUM COMPLETION TIME TRAJECTORY AND CHARGING OPTIMIZATION ALGORITHM

In this section, we propose the Minimum Completion Time Trajectory and Charging Optimization (MinTime-TCO) algorithm that solves the problem in (11) using a BCD approach. In particular, the problem in (11) is divided into two subproblems: the charging optimization and the hovering position optimization subproblems, which are solved in turn until convergence. In the first subproblem, we propose the Maximum Charge Rate Search (MCRS) algorithm to determine the optimal charging energies $\{e_j\}_{j=1}^J$ under fixed hovering positions $\{\mathbf{u}_j\}_{j=1}^J$. The optimality of the MCRS algorithm is then shown in Section VI. In the second subproblem, we propose the Hovering Position Optimization (HPO) algorithm, which instead optimizes the hovering positions $\{\mathbf{u}_j\}_{j=1}^J$ under fixed charging energies $\{e_j\}_{j=1}^J$. The HPO algorithm employs a successive convex approximation (SCA) approach to address the non-convexity of the objective and constraints. Thus, this method is referred to as the HPO-SCA algorithm.

A. Subproblem I: Charging Optimization

In this subsection, we first determine the optimal charging strategy $\{e_j\}_{j=1}^J$ under fixed hovering positions $\{\mathbf{u}_j\}_{j=1}^J$. In this case, with a slight abuse of notation, we can express $E_{j+1}^{\text{in}}(\mathbf{e}, \{\mathbf{u}_{j'}\}_{j'=1}^J)$ and $E_j^{\text{out}}(\mathbf{e}, \{\mathbf{u}_{j'}\}_{j'=1}^J)$ as $E_{j+1}^{\text{in}}(\mathbf{e})$ and $E_j^{\text{out}}(\mathbf{e})$, respectively. The optimization subproblem can then be formulated as follows:

$$\min_{e_j, \forall j} \sum_{j=1}^{J} \max\left(\frac{D_j}{R_j(\mathbf{u}_j)}, \frac{e_j}{p_j^{\max}(\mathbf{u}_j)}\right)$$
(12a)

subject to $E_{j+1}^{\text{in}}(\mathbf{e}) \ge 0, \forall j,$ (12b)

$$E_j^{\text{out}}(\mathbf{e}) \le E, \,\forall j.$$
 (12c)

Notice that flight time and height restrictions are disregarded in this subproblem since the charging occurs only at the hovering positions, which are fixed in this problem.

To solve the charging optimization subproblem in (12), we present the MCRS algorithm that determines sequentially the minimum energy needed to reach each of the J hovering positions. In particular, in each iteration (say, iteration j), the MCRS algorithm checks if position j can be reached under the current charging solution. If not, the algorithm revisits the charging solution at the previous j - 1 positions and increases the charging energy at the position with the maximum charging rate until the energy required to reach position j is obtained. For ease of exposition, we first assume that the data upload time (i.e., $D_j/R_j(\mathbf{u}_j)$, $\forall j$) is negligible compared to the charging

time in the following discussions. However, this may not be true in general. In such cases, the data upload time can provide the UAV with the opportunity to charge without incurring additional completion time. This can be later incorporated by allowing the UAV to prioritize charging during these intervals.

Suppose that $e^{j,MCRS} \triangleq (e_1^{j,MCRS}, \ldots, e_J^{j,MCRS})$ is the charging solution obtained by the MCRS algorithm in iteration j. This solution should only be sufficient for the UAV to reach position j without energy depletion. For solution $e^{j,MCRS}$, we can define j^{full} to be the last position for which the UAV is fully charged upon departure, i.e.,

$$j^{\text{full}} \triangleq \max\left\{j' \in \{0, 1, \dots, j-1\} : E_{j'}^{\text{out}}(\mathbf{e}^{j, \text{MCRS}}) = E\right\},$$
(13)

where $E_{j'}^{\text{out}}(\mathbf{e})$ is the remaining energy upon departure from position j' under charging policy \mathbf{e} as defined on the LHS of (9). Notice that, since the UAV is assumed to be fully charged upon arrival at the initial position, we set $E_0^{\text{out}}(\mathbf{e}) = E$. Hence, $j^{\text{full}} = 0$ in the initial iteration. Moreover, let

$$\Delta E_{j+1}^{\text{req}}(\mathbf{e}^{j,\text{MCRS}}) \triangleq \frac{\|\mathbf{u}_j - \mathbf{u}_{j+1}\|}{V} P_1 - E_j^{\text{out}}(\mathbf{e}^{j,\text{MCRS}})$$
(14)

be the minimum additional energy required for the UAV to reach position j + 1. Then, in iteration j + 1, the MCRS algorithm chooses to charge the required energy $\Delta E_{j+1}^{\text{req}}(\mathbf{e}^{j,\text{MCRS}})$ at the position with the maximum charge rate between $j^{\text{full}} + 1$ and j, i.e., the position

$$j^{\max} = \arg\max_{j' \in \{j^{\text{full}} + 1, \dots, j\}} p_{j'}^{\max}(\mathbf{u}_{j'}).$$
(15)

Please note that charging at positions before $j^{\text{full}} + 1$ would not be possible since this would violate the battery constraint at position j^{full} . By charging at position j^{\max} , the charging solution can be updated as

$$e_{j^{\max}}^{j+1,\text{MCRS}} = e_{j^{\max}}^{j,\text{MCRS}} + \frac{p_{j^{\max}}^{\max}(\mathbf{u}_{j^{\max}})}{p_{j^{\max}}^{\max}(\mathbf{u}_{j^{\max}}) - P_{0}}$$

$$\cdot \min\left\{\Delta E_{j+1}^{\text{req}}(\mathbf{e}^{j,\text{MCRS}}), \left\{E - E_{j'}^{\text{out}}(\mathbf{e}^{j,\text{MCRS}})\right\}_{j'=j^{\max}}^{j}\right\}.$$
(16)

The ratio $\frac{p_{j\max}^{\max}(\mathbf{u}_{j\max}) - P_0}{p_{j\max}^{\max}(\mathbf{u}_{j\max}) - P_0}$ accounts for the fact that energy is also consumed due to hovering during the energy charging time. The minimization in the second term reflects the fact that the charging energy at position j^{\max} is limited by the remaining battery capacity of all positions from j^{\max} to j. If $\Delta E_{j+1}^{\operatorname{req}}(\mathbf{e}^{j,\operatorname{MCRS}}) \leq \min\{E - E_{j'}^{\operatorname{out}}(\mathbf{e}^{j,\operatorname{MCRS}})\}_{j'=j^{\max}}^{j}$, then the required energy to reach position j + 1 is obtained without exceeding the battery capacity at later positions and, thus, we are done. If not, this implies that the battery at one of the positions from j^{\max} to j is full and the required energy is not yet fully obtained. In this case, we replace j^{full} with the index of this position (i.e., $j^{\operatorname{most}} \triangleq \arg\min_{j' \in \{j, \dots, j^{\max}\}} \{E - E_{j'}^{\operatorname{out}}(\mathbf{e}^{j,\operatorname{MCRS}})\}$), update the required energy $\Delta E_{j+1}^{\operatorname{req}}(\mathbf{e}^{j,\operatorname{MCRS}})$ using the new $\mathbf{e}^{j,\operatorname{MCRS}}$, and repeat the process again. The process is continued until the minimum required energy to reach position j + 1 is obtained. In the following, we provide an example to illustrate the procedures of the MCRS algorithm.



Fig. 2. An example of the MCRS algorithm.

Example: Suppose that there are 4 hovering positions with charge rates $p_1^{\max}(\mathbf{u}_1) = 100 \text{ W}, p_2^{\max}(\mathbf{u}_2) = 130 \text{ W}, p_3^{\max}(\mathbf{u}_3) =$ 110 W, $p_4^{\text{max}}(\mathbf{u}_4) = 140$ W. The battery capacity is E = 2000 J and is assumed to be full upon arrival at position 1. Initially, we set $e^{1,MCRS}$ to be an all-zero vector, i.e., $e^{1,MCRS} = 0$. Suppose that the remaining energy upon departure from position 1 is $E_1^{\text{out}}(\mathbf{e}^{1,\text{MCRS}}) = 200 \text{ J}$ and the minimum additional energy required to reach position 2 is $\Delta E_2^{\text{req}}(\mathbf{e}^{1,\text{MCRS}}) = 600 \text{ J}.$ In this case, the MCRS algorithm will update the charging energy at position 1 by letting $e_1^{2,\text{MCRS}} = \frac{100}{100-P_0}600$, as illustrated in Fig. 2(a). The charging energies at other positions remain to be 0. Notice that the energy obtained after the first iteration is only sufficient to reach position 2 and, thus, $E_2^{\text{in}}(\mathbf{e}^{2,\text{MCRS}}) = 0$. Moreover, suppose that the minimum additional energy required to reach position 3 is $\Delta E_3^{\text{req}}(\mathbf{e}^{2,\text{MCRS}}) =$ 1200 J. Then, since $p_2^{\max}(\mathbf{u}_2) > p_1^{\max}(\mathbf{u}_1)$, the MCRS algorithm will choose to charge the additional energy at position 2 and update the charging energy at position 2 as $e_2^{3,\mathrm{MCRS}} =$ $e_2^{2,\text{MCRS}} + \frac{130}{130-P_0}$ 1200, as illustrated in Fig. 2(b). With the updated solution e^{3,MCRS}, the remaining energy upon departure from position 2 is $E_2^{\text{out}}(\mathbf{e}^{3,\text{MCRS}}) = 1200 \text{ J}$ and the remaining energy upon arrival at position 3 is $E_3^{\text{in}}(\mathbf{e}^{3,\text{MCRS}}) = 0$. Finally, suppose that the minimum additional energy required to reach position 4 is $\Delta E_4^{\text{req}}(\mathbf{e}^{3,\text{MCRS}}) = 1000 \text{ J. Since } p_2^{\text{max}}(\mathbf{u}_2) >$

Algorithm 1: Maximum Charge Rate Search (MCRS) Algorithm.

1 I 1	nitialize: $\mathbf{e}^{1,\mathrm{MCRS}} \leftarrow \{0,\ldots,0\}, 1^{\mathrm{full}} \leftarrow 0, \mathcal{F} \leftarrow \emptyset;$		
2 for $j = 1$ to J do			
3	$\mathbf{e}^{j, ext{MCRS}} \leftarrow \mathbf{e}^{j-1, ext{MCRS}};$		
4	$j^{\text{full}} \leftarrow (j-1)^{\text{full}};$		
5	$(\mathbf{e}^{j,\mathrm{MCRS}},\mathcal{F},j^{\mathrm{full}}) \leftarrow$		
	<code>Free_Charge</code> ($\mathbf{e}^{j,\mathrm{MCRS}},\mathcal{F}\cup\{j\}$);		
6	while $\Delta E_{j+1}^{\text{req}}(\mathbf{e}^{j,\text{MCRS}}) > 0$ do		
7	$j^{\max} \leftarrow \arg\max_{j' \in \{j^{\text{full}}+1,\ldots,j\}} p_{j'}^{\max}(\mathbf{u}_{j'});$		
8	$j^{\text{most}} \leftarrow$		
	$\arg\min_{j' \in \{j^{\max}, \dots, j\}} E - E_{j'}^{\text{out}}(\mathbf{e}^{j, \text{MCRS}});$		
9	$e^{\max} \leftarrow E - E_{j^{\text{most}}}^{\text{out}}(\mathbf{e}^{j,\text{MCRS}});$		
10	$e_{j^{\max}}^{j,\mathrm{MCRS}} \leftarrow e_{j^{\max}}^{j,\mathrm{MCRS}} +$		
	$\min(e^{\max}, \Delta E_{j+1}^{\operatorname{req}}(\mathbf{e}^{j,\operatorname{MCRS}})) \frac{p_{j\max}^{\max}(\mathbf{u}_{j\max})}{p_{j\max}^{\max}(\mathbf{u}_{j\max}) - P_0};$		
11	if $E_{j^{\text{most}}}^{\text{out}}(\mathbf{e}^{j,\text{MCRS}}) = E$ then		
12	$ j^{\text{full}} \leftarrow j^{\text{most}};$		
13	end		
14	end		
15 ei	nd		

1 Function Free_Charge ($\mathbf{e}^{j,\mathrm{MCRS}},\mathcal{F}$) while $\mathcal{F} \neq \overline{\emptyset}$ and $\Delta E_{j+1}^{\mathrm{req}}(\mathbf{e}^{j,\mathrm{MCRS}}) > 0$ do 2 $\begin{array}{l} j^{\text{free}} \leftarrow \min \mathcal{F}; \\ e^{\max} \leftarrow E - E_{j^{\text{free}}}^{\text{out}}(\mathbf{e}^{j,\text{MCRS}}); \\ e^{\text{free}} \leftarrow p_{j^{\text{free}}}^{\max}(\mathbf{u}_{j}) \frac{D_{j^{\text{free}}}}{R_{j^{\text{free}}}(\mathbf{u}_{j^{\text{free}}})} - e_{j^{\text{free}}}^{j,\text{MCRS}}; \\ e_{j^{\text{free}}}^{j,\text{MCRS}} \leftarrow e_{j^{\text{free}}}^{j,\text{MCRS}} \leftarrow e_{j^{\text{free}}}^{j,\text{MCRS}} + e_{j^{\text{free}}}^{j,\text{MC$ 3 4 5 6 $e_{j^{\text{free}}}^{j,\text{MCRS}} + \min(e^{\max}, \Delta E_{j+1}^{\text{req}}(\mathbf{e}^{j,\text{MCRS}}), e^{\text{free}});$ if $E_{j^{\text{free}}}^{\text{out}}(\mathbf{e}^{j,\text{MCRS}}) = E$ then 7 $j^{\text{full}} \leftarrow j^{\text{free}};$ 8 $\mathcal{F} \leftarrow \mathcal{F} \setminus \{j^{\text{free}}\};$ 9
$$\begin{split} & \text{if } p_{j^{\text{free}}}^{\max}(\mathbf{u}_j) \frac{D_{j^{\text{free}}}}{R_{j^{\text{free}}}(\mathbf{u}_{j^{\text{free}}})} = e_{j^{\text{free}}}^{j,\text{MCRS}} \text{ then} \\ & \mid \mathcal{F} \leftarrow \mathcal{F} \setminus \{j^{\text{free}}\}; \end{split}$$
10 11 12 end 13 end 14 return ($\mathbf{e}^{j,\mathrm{MCRS}},\mathcal{F},j^{\mathrm{full}}$); 15

 $\max\{p_1^{\max}(\mathbf{u}_1), p_3^{\max}(\mathbf{u}_3)\}$, the MCRS algorithm will prioritize charging at position 2 but can only obtain part of the required energy at this position since the remaining battery capacity upon departure from position 2 is only E-1200=800 J. Hence, the charging energy at position 2 is only updated as $e_2^{4,\mathrm{MCRS}} = e_3^{3,\mathrm{MCRS}} + \frac{130}{130-P_0}800$, as illustrated in Fig. 2(c). Consequently, the remaining energy upon arrival at position 3 is now $E_3^{\mathrm{in}}(\mathrm{e}^{3,\mathrm{MCRS}}) = 800$ J. After charging 800 J of the required 1000 J energy at position 2, the remaining 200 J that is required can only be charged at position 3, as illustrated in Fig. 2(d). Hence, we have $e_3^{4,\mathrm{MCRS}} = \frac{110}{110-P_0}200$.

The pseudo-code of the MCRS algorithm is summarized in Algorithm 1, and its optimality proof is provided in Section VI. In Algorithm 1, we include the free charge procedure to account

for the energy that can be obtained without additional cost during the data upload time. Specifically, we initialize the charging solution as $e^{1,MCRS} = 0$, set 1^{full} to 0 (since the UAV is assumed to be fully charged upon dispatch), and set \mathcal{F} to be an empty set. Here, \mathcal{F} is used to store the indices of positions where free charging opportunities during data upload times have not yet been fully utilized. In the main body of the algorithm, the positions are visited sequentially, from 1 to J, to check whether the remaining energy of the UAV is sufficient to reach each position. If not, the required energy is charged at the position with the maximum charge rate. In each iteration, $e^{j,\mathrm{MCRS}}$ and j^{full} are initialized with the values from the previous iteration, i.e., $e^{j-1,MCRS}$ and $(j-1)^{full}$. Then, the Free_Charge procedure is applied to utilize the available charging energy during data upload times up to position j. After fully exploiting free charging opportunities, we proceed through the while loop to find the position j^{\max} with the maximum charge rate and charge the required energy at this position until the battery capacity is reached at one of the positions from j^{max} to j. The process is repeated if the required energy $\Delta E_{j+1}^{\text{req}}(\mathbf{e}^{j,\text{MCRS}})$ has not yet been fully obtained. The Free_Charge function charges at the earliest available position in the set \mathcal{F} until the free charge amount is exhausted at that position or the required energy is obtained.

B. Subproblem II: Hovering Position Optimization

In this subsection, we aim to determine the optimal hovering positions $\{\mathbf{u}_j\}_{j=1}^J$ under fixed charging strategy $\{e_j\}_{j=1}^J$ (or e). In this case, we can express $E_{j+1}^{\text{in}}(\mathbf{e}, \{\mathbf{u}_{j'}\}_{j'=1}^J)$ and $E_j^{\text{out}}(\mathbf{e}, \{\mathbf{u}_{j'}\}_{j'=1}^J)$ as $E_{j+1}^{\text{in}}(\{\mathbf{u}_j\})$ and $E_j^{\text{out}}(\{\mathbf{u}_j\})$, respectively, where $\{\mathbf{u}_j\}$ is used as a concise notation for $\{\mathbf{u}_{j'}\}_{j'=1}^J$. Then, the optimization subproblem can be formulated as

$$\min_{\mathbf{u}_{j},\forall j} \sum_{j=1}^{J} \max\left(\frac{D_{j}}{R_{j}(\mathbf{u}_{j})}, \frac{e_{j}}{p_{j}^{\max}(\mathbf{u}_{j})}\right) + \sum_{j=1}^{J-1} \frac{\|\mathbf{u}_{j} - \mathbf{u}_{j+1}\|}{V}$$
(17a)

subject to
$$E_{j+1}^{\text{in}}({\mathbf{u}_j}) \ge 0, \forall j,$$
 (17b)

$$E_j^{\text{out}}(\{\mathbf{u}_j\}) \le E, \,\forall j,\tag{17c}$$

$$H_{\min} \le u_{j,3} \le H_{\max}, \,\forall j. \tag{17d}$$

Notice that modifying the hovering positions will simultaneously alter the charge and data transmission rates (and, thus, the charge and data upload times). The distance between hovering positions will also change, further affecting the total flight time. By (10) and the definition of $E_j^{\text{out}}(\{\mathbf{u}_j\})$, we can rewrite the problem as

$$\min_{\mathbf{u}_{j},\forall j} \sum_{j=1}^{J} \max\left(\frac{D_{j}}{R_{j}(\mathbf{u}_{j})}, \frac{e_{j}}{p_{j}^{\max}(\mathbf{u}_{j})}\right) + \sum_{j=1}^{J-1} \frac{\|\mathbf{u}_{j} - \mathbf{u}_{j+1}\|}{V}$$
(18a)

subject to
$$E_j^{\text{out}}({\mathbf{u}_j}) - \frac{\|\mathbf{u}_{j+1} - \mathbf{u}_j\|}{V} P_1 \ge 0, \forall j,$$
 (18b)

$$E_{j-1}^{\text{out}}(\{\mathbf{u}_j\}) + e_j - \frac{\|\mathbf{u}_j - \mathbf{u}_{j-1}\|}{V} P_1$$
$$-\max\left(\frac{D_j}{R_j(\mathbf{u}_j)}, \frac{e_j}{p_j^{\max}(\mathbf{u}_j)}\right) P_0 \le E, \,\forall j, \quad (18c)$$

$$H_{\min} \le u_{j,3} \le H_{\max}, \,\forall j. \tag{18d}$$

To provide more flexibility for the optimization of $\{\mathbf{u}_j\}_{j=1}^J$, we propose to remove the maximum charging constraint in (18c), but instead account for the battery capacity limit by replacing $E_j^{\text{out}}(\{\mathbf{u}_j\})$ in (18b) with the equation

$$\min\left\{E, E_{j-1}^{\text{out}}(\{\mathbf{u}_j\}) + e_j - \frac{\|\mathbf{u}_j - \mathbf{u}_{j-1}\|}{V}P_1 - \max\left(\frac{D_j}{R_j(\mathbf{u}_j)}, \frac{e_j}{p_j^{\max}(\mathbf{u}_j)}\right)P_0\right\}$$
(19)

By doing so, we ensure that the remaining energy upon departure from position j is not over-estimated even after removing the constraint in (18c). Then, by introducing the auxiliary variables b_j , for all j, the problem can be reduced to the following:

$$\min_{\mathbf{u}_{j}, b_{j}, \forall j} \sum_{j=1}^{J} b_{j} + \sum_{j=1}^{J-1} \frac{\|\mathbf{u}_{j} - \mathbf{u}_{j+1}\|}{V}$$
(20a)

subject to
$$\tilde{E}_j^{\text{out}}(\{\mathbf{u}_j\}) - \frac{\|\mathbf{u}_{j+1} - \mathbf{u}_j\|}{V} P_1 \ge 0, \, \forall j, \quad (20b)$$

$$\max\left(\frac{D_j}{R_j(\mathbf{u}_j)}, \frac{e_j}{p_j^{\max}(\mathbf{u}_j)}\right) \le b_j, \,\forall j, \qquad (20c)$$

$$H_{\min} \le u_{j,3} \le H_{\max}, \,\forall j, \tag{20d}$$

where

$$\tilde{E}_{j}^{\text{out}}(\{\mathbf{u}_{j}\}) = \min\left\{E, \ \tilde{E}_{j-1}^{\text{out}}(\{\mathbf{u}_{j}\}) + e_{j} - \frac{\|\mathbf{u}_{j} - \mathbf{u}_{j-1}\|}{V}P_{1} - b_{j}P_{0}\right\}.$$
 (21)

It is worthwhile to remark that any solution that is feasible for the original subproblem in (18) is also feasible for the relaxed problem in (20). This is because, in (18), the charging energy solution is required to satisfy the battery constraint in (18c) and, thus, the remaining energy upon departure from any position would not exceed E. In this case, we have $\tilde{E}_j^{\text{out}}(\{\mathbf{u}_j\}) = E_j^{\text{out}}(\{\mathbf{u}_j\})$ and, thus, the constraint in (20b) becomes equivalent to (18b). Therefore, the resulting objective value of the problem in (20) should be no larger than that of the problem in (18). Moreover, for any solution of $\{\mathbf{u}_j\}_{j=1}^J$ that is feasible for the above problem, there exists a charging energy solution that satisfies the constraint in (12c) by reducing the charge amount e_j as

$$e_{j}^{\text{reduced}} \leftarrow e_{j} - \max\left\{\tilde{E}_{j-1}^{\text{out}}(\{\mathbf{u}_{j}\}) + e_{j} - \frac{\|\mathbf{u}_{j} - \mathbf{u}_{j-1}\|}{V}P_{1} - b_{j}P_{0} - E, 0\right\}.$$
(22)

Hence, we can ensure that the objective value is not increased in the next iteration.

The optimization problem in (20) is non-convex due to the constraint in (20c) and, thus, is difficult to solve efficiently. To address this issue, we adopt an SCA approach where the non-convex terms on the LHS of (20c) are replaced with their convex upper bounds. In particular, by replacing $R_j(\mathbf{u}_j)$ and $p_j^{\max}(\mathbf{u}_j)$ with their definitions in (4) and (6) (and also (5)), we can rewrite the constraint in (20c) for position j as

$$\frac{e_j}{\max_{i \in \mathcal{I}} \frac{\eta P^{\mathrm{L}} \exp(-\gamma \|\mathbf{a}_i - \mathbf{u}_j\|)}{(\zeta + \phi \|\mathbf{a}_i - \mathbf{u}_j\|)^2}} \le b_j,$$
(23)

$$\frac{D_j}{B\log\left(1 + \frac{P^{\operatorname{tr}}g_j(\mathbf{u}_j)}{BN_0}\right)} \le b_j.$$
(24)

Let $\mathbf{u}_{j}^{\text{iter}}$ be the coordinates of position j obtained in the previous iteration of the MinTime-TCO algorithm, and let

$$\mathbf{a}_{j}^{\text{iter}} = \underset{\mathbf{a} \in \{\mathbf{a}_{1}, \dots, \mathbf{a}_{I}\}}{\arg \max} \frac{\eta P^{\text{L}} \exp(-\gamma \|\mathbf{a} - \mathbf{u}_{j}^{\text{iter}}\|)}{(\zeta + \phi \|\mathbf{a} - \mathbf{u}_{j}^{\text{iter}}\|)^{2}}$$
(25)

be the coordinates of the corresponding HAP chosen to charge the UAV when it is at position $\mathbf{u}_{j}^{\text{iter}}$. In this case, we have

$$\frac{e_j}{\max_{i\in\{1,\dots,I\}}\frac{\eta P^{\mathrm{L}}\exp(-\gamma\|\mathbf{a}_i-\mathbf{u}_j\|)}{(\zeta+\phi\|\mathbf{a}_i-\mathbf{u}_j\|)^2}} \le \frac{e_j}{\frac{\eta P^{\mathrm{L}}\exp(-\gamma\|\mathbf{a}_j^{\mathrm{iter}}-\mathbf{u}_j\|)}{(\zeta+\phi\|\mathbf{a}_j^{\mathrm{iter}}-\mathbf{u}_j\|)^2}}$$
(26)

with equality when $\mathbf{u}_j = \mathbf{u}_j^{\text{iter}}$. Then, by replacing the LHS of (23) with its upper bound given above and by taking the logarithm on both sides, we have

$$\ln \frac{e_j}{\eta P^{\mathsf{L}}} + \gamma \| \mathbf{a}_j^{\text{iter}} - \mathbf{u}_j \| + 2\ln(\zeta + \phi \| \mathbf{a}_j^{\text{iter}} - \mathbf{u}_j \|) \le \ln b_j.$$
(27)

We can see that $2\ln(\zeta + \phi \|\mathbf{a}_j^{\text{iter}} - \mathbf{u}_j\|)$ is concave with respect to $\|\mathbf{a}_j^{\text{iter}} - \mathbf{u}_j\|$ and, thus, can be upper bounded by its first-order Taylor expansion, that is,

$$2\ln(\zeta + \phi \|\mathbf{a}_{j}^{\text{iter}} - \mathbf{u}_{j}\|) \leq 2\ln(\zeta + \phi \|\mathbf{a}_{j}^{\text{iter}} - \mathbf{u}_{j}^{\text{iter}}\|) + \frac{2\phi}{\zeta + \phi \|\mathbf{a}_{j}^{\text{iter}} - \mathbf{u}_{j}^{\text{iter}}\|} (\|\mathbf{a}_{j}^{\text{iter}} - \mathbf{u}_{j}\| - \|\mathbf{a}_{j}^{\text{iter}} - \mathbf{u}_{j}^{\text{iter}}\|).$$

$$(28)$$

Then, by further replacing the term $2\ln(\zeta + \phi ||\mathbf{a}_j^{\text{iter}} - \mathbf{u}_j||)$ in (27) with the right-hand-side (RHS) of (28), we obtain a convex constraint given by

$$\ln \frac{e_j}{\eta P^{\mathrm{L}}} + \gamma \|\mathbf{a}_j^{\mathrm{iter}} - \mathbf{u}_j\| + 2\ln(\zeta + \phi \|\mathbf{a}_j^{\mathrm{iter}} - \mathbf{u}_j^{\mathrm{iter}}\|) + \frac{2\phi}{\zeta + \phi \|\mathbf{a}_j^{\mathrm{iter}} - \mathbf{u}_j^{\mathrm{iter}}\|} (\|\mathbf{a}_j^{\mathrm{iter}} - \mathbf{u}_j\| - \|\mathbf{a}_j^{\mathrm{iter}} - \mathbf{u}_j^{\mathrm{iter}}\|) \leq \ln b_j.$$
(29)

The bound is tight at the point $\mathbf{u}_j = \mathbf{u}_j^{\text{iter}}$ and thus the solution $\mathbf{u}_j^{\text{iter}}$ obtained in the previous iteration is also feasible under this constraint. Also, by (26) and (28), it follows that any value of $\mathbf{u}_j^{\text{iter}}$ that is feasible under (29) is also feasible under the original constraint in (27) (or, equivalently, (23)).

Moreover, by introducing the auxiliary variable c_j and by the definition of $g_j(\cdot)$ given in (3), the constraint in (24) can be written as

$$\frac{D_j}{b_j} \le B \log\left(1 + \frac{P^{\text{tx}}c_j}{BN_0}\right),\tag{30}$$

$$c_j \le \Phi_j^{\text{LoS}}(\mathbf{u}_j) \frac{\rho_0^{\text{LoS}}}{\|\mathbf{u}_j - \mathbf{s}_j\|^{\alpha^{\text{LoS}}}}.$$
(31)

The constraint in (30) is convex, but the constraint in (31) is not. To further cope with the non-convexity of (31), we first take the logarithm on both sides. Then, by the definition of $\Phi_j^{\text{LoS}}(\mathbf{u}_j)$ in (2) and by introducing another auxiliary variable d_j , we can rewrite (31) as

$$\ln c_j \le \ln \frac{1}{1 + C_2 \exp(C_1 C_2) d_j} + \ln \frac{\rho_0^{\text{LoS}}}{\|\mathbf{u}_j - \mathbf{s}_j\|^{\alpha^{\text{LoS}}}}, \quad (32)$$

$$\exp\left[-C_1\frac{180}{\pi}\arcsin\left(\frac{u_{j,3}}{\|\mathbf{u}_j - \mathbf{s}_j\|}\right)\right] \le d_j.$$
(33)

By reorganizing the terms, (32) can be written as

$$\ln c_{j} + \ln \left(1 + C_{2} \exp(C_{1}C_{2})d_{j}\right) + \ln \|\mathbf{u}_{j} - \mathbf{s}_{j}\|^{\alpha^{\text{LoS}}} \leq \ln \rho_{0}^{\text{LoS}}$$
(34)

Let c_j^{iter} be the solution of c_j obtained in the previous iteration. By the fact that $\log x$ is concave with respect to x, for x > 0, we can upper bound each term on the LHS of (35) by their first order Taylor approximations, which results in the following convex constraint

$$\ln c_{j}^{\text{iter}} + \frac{1}{c_{j}^{\text{iter}}} (c_{j} - c_{j}^{\text{iter}}) + \ln \left(1 + C_{2} \exp(C_{1}C_{2})d_{j}^{\text{iter}}\right) + \frac{C_{2} \exp(C_{1}C_{2})}{1 + C_{2} \exp(C_{1}C_{2})d_{j}^{\text{iter}}} (d_{j} - d_{j}^{\text{iter}}) + \ln \|\mathbf{u}_{j}^{\text{iter}} - \mathbf{s}_{j}\|^{\alpha^{\text{LoS}}} + \frac{\alpha^{\text{LoS}}}{\|\mathbf{u}_{j}^{\text{iter}} - \mathbf{s}_{j}\|} (\|\mathbf{u}_{j} - \mathbf{s}_{j}\| - \|\mathbf{u}_{j}^{\text{iter}} - \mathbf{s}_{j}\|) \le \ln \rho_{0}^{\text{LoS}}.$$
(35)

Similarly, to address the non-convexity of (33), we further introduce the auxiliary variables f_j and h_j so that (33) can be rewritten as

$$\exp\left(-C_1\frac{180}{\pi}f_j\right) \le d_j,\tag{36}$$

$$f_j \le \arcsin(h_j),\tag{37}$$

$$h_j \le \frac{u_{j,3}}{\|\mathbf{u}_j - \mathbf{s}_j\|},\tag{38}$$

$$h_j \ge 0. \tag{39}$$

Notice that (36) is convex, but (37) and (38) are not. By the fact that $\arcsin(h_j)$ is convex with respect to h_j , for $h_j \ge 0$, we can replace the RHS of (37) with its first-order Taylor approximation around the initial solution h_j^{iter} (i.e., the solution of h_j obtained in the previous iteration). This yields an approximate convex constraint given by

$$f_j \le \arcsin(h_j^{\text{iter}}) + \frac{1}{\sqrt{1 + (h_j^{\text{iter}})^2}} (h_j - h_j^{\text{iter}}).$$
 (40)

Algorithm 2: Minimum Completion Time Trajectory and Charging Optimization (MinTime-TCO) Algorithm.

1 Initialize: Set $\mathbf{u}_j \leftarrow (x_j^{\mathrm{S}}, y_j^{\mathrm{S}}, H_{\max})$, for $j = 1, \ldots, J$;2 set $t \leftarrow 0$ and $\mathrm{Obj}^{(0)} \leftarrow \infty$;3 repeat4999<

By reorganizing the constraint in (38) and by taking the logarithm on both sides, we have

$$\ln h_j + \ln \|\mathbf{u}_j - \mathbf{s}_j\| \le \ln u_{j,3}.$$
(41)

By the fact that $\ln x$ is concave, we can replace the LHS of (41) with their first-order Taylor approximations to obtain an approximate convex constraint given by

$$\ln h_{j}^{\text{iter}} + \frac{1}{h_{j}^{\text{iter}}} (h_{j} - h_{j}^{\text{iter}}) + \ln \|\mathbf{u}_{j}^{\text{iter}} - \mathbf{s}_{j}\| + \frac{1}{\|\mathbf{u}_{j}^{\text{iter}} - \mathbf{s}_{j}\|} (\|\mathbf{u}_{j} - \mathbf{s}_{j}\| - \|\mathbf{u}_{j}^{\text{iter}} - \mathbf{s}_{j}\|) \le \ln u_{j,3}.$$
(42)

By replacing the constraints in (20c) with their convex approximations in (29), (30), (35), (36), (39), (40) and (42), we can approximate the problem in (17) with the following convex optimization problem

$$\min_{\mathbf{u}_{j}, b_{j}, c_{j}, d_{j}, f_{j}, h_{j}, \forall j} \sum_{j=1}^{J} b_{j} + \sum_{j=1}^{J-1} \frac{\|\mathbf{u}_{j} - \mathbf{u}_{j+1}\|}{V}$$
(43a)

subject to (20b), (20d), (29), (30), (35),

$$(36), (39), (40), (42), \forall j.$$
 (43b)

The problem is convex and can be solved by off-the-shelf solvers, such as CVX [32]. By adopting the interior point method, the computational complexity required to solve the convex optimization problem in (43) to ϵ accuracy is $\mathcal{O}((8J)^3 \ln(1/\epsilon))$ [33], [34]. The above optimization procedure is referred to as the HPO-SCA algorithm.

Summary of the MinTime-TCO Algorithm: The proposed MinTime-TCO algorithm solves the problem in (11) using a BCD approach where the charging energies $\{e_j\}_{j=1}^J$ and hovering positions $\{\mathbf{u}\}_{j=1}^J$ are optimized in turn until convergence. In each iteration, the optimal charging energies are determined by the MCRS algorithm, and the UAV's charging positions are optimized by the HPO-SCA algorithm (i.e., by solving the convex optimization problem in (43)). The MinTime-TCO algorithm is summarized in Algorithm 2. Here, $Obj^{(t)}$ refers to the objective value in (11) obtained during the *t*-th iteration. The convergence of the MinTime-TCO algorithm is shown below.

Theorem 1: The sequence of objective values obtained by the MinTime-TCO algorithm is monotonically non-increasing and thus converges.

Proof: Let $\{e_j[t]\}_{j=1}^J$ and $\{\mathbf{u}_j[t]\}_{j=1}^J$ be the solutions obtained at the end of iteration t and let $J(\{e_j\}_{j=1}^J, \{\mathbf{u}_j\}_{j=1}^J)$ be the objective value in (11a) as a function of the variables $\{e_j\}_{j=1}^J$ and $\{\mathbf{u}_j\}_{j=1}^J$. Then, in iteration t+1, the solution $\{e_j[t+1]\}_{j=1}^J$ in Subproblem I is first obtained given $\{\mathbf{u}_j[t]\}_{j=1}^J$ by applying the MCRS algorithm, which yields the optimal solution to the problem in (12). Hence, it must hold that $J(\{e_j[t+1]\}_{j=1}^J, \{\mathbf{u}_j[t]\}_{j=1}^J) \leq$ $J(\{e_j[t]\}_{j=1}^J, \{\mathbf{u}_j[t]\}_{j=1}^J)$. Then, in Subproblem II, the solution $\{\mathbf{u}_j[t+1]\}_{j=1}^J$ given $\{e_j[t+1]\}_{j=1}^J$ is obtained by solving the optimization problem in (43). Notice that (43) is a convex approximation of the problem in (20). The convex approximations are tight at the point $\{\mathbf{u}_j[t]\}_{j=1}^J$ and, thus, the solutions obtained in iteration t are also feasible under the convex constraints in (43). Hence, the solution $\{\mathbf{u}_{j}[t+1]\}_{j=1}^{J}$ along with the corresponding charging solution $\{e_j^{\text{reduced}}[t+1]\}_{j=1}^J$ described in (22) further reduces the objective value, i.e., $J(\{e_j^{\text{reduced}}[t+1]\}_{j=1}^J, \{\mathbf{u}_j[t+1]\}_{j=1}^J) \leq J$ $J(\{e_j[t+1]\}_{j=1}^J, \{\mathbf{u}_j[t]\}_{j=1}^J)$. By combining the above arguments and by replacing $\{e_j[t+1]\}_{j=1}^J$ with the updated solution $\{e_j^{\text{reduced}}[t+1]\}_{j=1}^J$, we have $J(\{e_j[t+1]\}_{j=1}^J, \{\mathbf{u}_j[t+1]\}_{j=1}^J, \{\mathbf{u}_j[t+1]\}_{j=$ $1]\}_{j=1}^{J} \leq J(\{e_j[t]\}_{j=1}^{J}, \{\mathbf{u}_j[t]\}_{j=1}^{J}), \text{ i.e., the sequence of ob$ jective values $\{J(\{e_j[t]\}_{j=1}^J, \{\mathbf{u}_j[t]\}_{j=1}^J)\}_{t=1}^\infty$ is monotonically non-increasing. Moreover, since the objective function is bounded below, the sequence must converge. \square

V. APPROXIMATE HOVERING POSITION OPTIMIZATION BY DYNAMIC PROGRAMMING

In the previous sections, we adopted the SCA method to yield a tractable solution for the non-convex hovering position optimization subproblem in (17). Even though SCA already reduces the complexity of the problem, the need for iterative numerical methods, such as the interior point method, to solve the relaxed convex optimization problem in each iteration can still be time-consuming (c.f., Table II). Therefore, in this section, we propose a low-complexity approach using the DP principle to search across a set of potential grid points above each sensor. The complexity of the algorithm can be adjusted by the choice of the number of grid points, with a manageable tradeoff in performance. More importantly, when alternating with the energy charging optimization problem, the number of iterations required to converge when using DP is much smaller than that when using SCA, as we show later in our experiments. The resulting method is called the HPO-DP algorithm.

Specifically, to find the optimal hovering positions $\{\mathbf{u}_j\}_{j=1}^J$ using DP, we first discretize the feasible hovering positions above each sensor node j to yield the set

$$\mathcal{U}_j = \left\{ (s_{j,1} + m_1 \delta, s_{j,2} + m_2 \delta, m_3 \delta) : \\ m_1, m_2, m_3 \text{ are integers,} \\ R_j(s_{j,1} + m_1 \delta, s_{j,2} + m_2 \delta, m_3 \delta) \ge R^{\text{th}} \right.$$

and
$$H_{\min} \le m_3 \delta \le H_{\max}$$
 (44)

Note that \mathcal{U}_j is a 3-dimensional grid of feasible hovering points above sensor j with transmission rate greater than the threshold R^{th} to ensure reliable communication between sensor j and the UAV. The number of feasible hovering positions is denoted by $L_j = |\mathcal{U}_j|$.

Let $\mathbf{u}_{j,l}$ be the coordinates of the *l*-th point in \mathcal{U}_j and let $\mathcal{V}_{j,l}$ be the set of indices of points in \mathcal{U}_{j-1} that has sufficient energy to reach point $\mathbf{u}_{j,l}$ under solution e, i.e.,

$$\mathcal{V}_{j,l} = \left\{ l' \in \{1, \dots, L_{j-1}\} : \\ E_{j-1,l'}^{\text{out}}(\mathbf{e}) - \frac{\|\mathbf{u}_{j,l} - \mathbf{u}_{j-1,l'}\|}{V} P_1 \ge 0 \right\}.$$
(45)

Then, for each point $\mathbf{u}_{j,l}$, we can select a path arriving from one of the points in $\mathcal{V}_{j,l}$ that yields the minimum accumulated flight time up to $\mathbf{u}_{j,l}$. The selection can be done recursively from points in \mathcal{U}_1 to points in \mathcal{U}_J . When arriving at the points in \mathcal{U}_j , the accumulated flight time to reach each point $\mathbf{u}_{j,l} \in \mathcal{U}_j$ is updated using the following Bellman equation

$$T_{j,l} = \min_{l' \in \mathcal{V}_{j,l}} \left\{ T_{j-1,l'} + \frac{\|\mathbf{u}_{j,l} - \mathbf{u}_{j-1,l'}\|}{V} \right\} + \max\left(\frac{D_j}{R_j(\mathbf{u}_{j,l})}, \frac{e_j}{p_j^{\max}(\mathbf{u}_{j,l})}\right)$$
(46)

for j > 1, where $T_{1,l} = \max(\frac{D_j}{R_j(\mathbf{u}_{j,l})}, \frac{e_j}{p_j^{\max}(\mathbf{u}_{j,l})})$. Similarly, the remaining energy upon departure from position $\mathbf{u}_{j,l} \in \mathcal{U}_j$ can be updated as

$$E_{j,l}^{\text{out}}(\mathbf{e}) = \min\left\{E, E_{j-1,l_{j,l}^{\min}}^{\text{out}}(\mathbf{e}) - \frac{\|\mathbf{u}_{j,l} - \mathbf{u}_{j-1,l_{j,l}^{\min}}\|}{V}P_{1} + e_{j} - \max\left(\frac{D_{j}}{R_{j}(\mathbf{u}_{j,l})}, \frac{e_{j}}{p_{j}^{\max}(\mathbf{u}_{j,l})}\right)P_{0}\right\}$$
(47)

where $E_{1,l}^{\text{out}}(\mathbf{e}) = \min(E, E + e_1 - \max(\frac{D_1}{R_1(\mathbf{u}_{1,l})}, \frac{e_1}{p_1^{\max}(\mathbf{u}_{1,l})})P_0)$ and

$$l_{j,l}^{\min} = \underset{l' \in \mathcal{V}_{j,l}}{\arg\min} \left\{ T_{j-1,l'} + \frac{\|\mathbf{u}_{j,l} - \mathbf{u}_{j-1,l'}\|}{V} \right\}$$
(48)

is the index of the point in $\mathcal{V}_{j,l}$ that yields the minimum accumulated flight time to reach $\mathbf{u}_{j,l}$. By performing the above computations recursively from points in \mathcal{U}_1 to points in \mathcal{U}_J , we can eventually find the set of accumulated flight times $T_{J,l}$, for all $l \in \mathcal{U}_J$. Then, the point in \mathcal{U}_J with the minimum accumulated flight time is selected, and the desired UAV trajectory can be obtained by backtracking through the previous positions. The HPO-DP algorithm is summarized in Algorithm 3.

Specifically, in Line 1, we initialize $T_{j,l}$ to ∞ and set $E_{j,l}^{\text{out}}(\mathbf{e})$ and $l_{j,l}^{\min}$ to -1, indicating that these values have not been calculated. Then, in Lines 2-5 we compute the flight time and remaining energy for all points above sensor 1. Subsequently, in Lines 6-15, we compute the minimum accumulated flight time $T_{j,l}$ to reach each discretized point in \mathcal{U}_j , record the preceding point $l_{j,l}^{\min}$ on the trajectory, and update the remaining energy upon departure $E_{j,l}^{\text{out}}(\mathbf{e})$. If no points above the previous **Algorithm 3:** Hovering Position Optimization by Dynamic Programming (HPO-DP) Algorithm.

1 Initialize: Set
$$T_{j,l} \leftarrow \infty$$
, $l_{j,l}^{\min} \leftarrow -1$, and
 $E_{j,l}^{\text{out}}(e^J) \leftarrow -1$, for all j and l ;
2 for $l = 1$ to L_1 do
3 Set $T_{1,l} \leftarrow \max\left(\frac{D_j}{R_j(\mathbf{u}_{j,l})}, \frac{e_j}{p_j^{\max}(\mathbf{u}_{j,l})}\right)$;
4 Set $E_{1,l}^{\text{out}}(\mathbf{e}) \leftarrow \min(E, E + e_1 - \max\left(\frac{D_1}{R_1(\mathbf{u}_{1,l})}, \frac{e_1}{p_1^{\max}(\mathbf{u}_{1,l})}\right)P_0)$;
5 end
6 for $j = 2$ to J do
7 for $l = 1$ to L_j do
8 Set $\mathcal{V}_{j,l} \leftarrow \{l' \in \{1, \dots, L_{j-1}\} | E_{j-1,l'}^{\text{out}}(\mathbf{e}) - \frac{||\mathbf{u}_{j,l} - \mathbf{u}_{j-1,l'}||}{V}P_1 \ge 0\}$;
9 lif $\mathcal{V}_{j,l} \neq \emptyset$ then
10 calculate $T_{j,l}$ through equation (46);
11 calculate $L_{j,l}^{\text{out}}$ through equation (48);
12 calculate $E_{j,l}^{\text{out}}$ (e) through equation (47);
13 end
14 end
15 end
16 $l^* \leftarrow \arg\min_{l \in \{1, \dots, L_J\}} T_{J,l}$;
17 for $j = J$ to 1 do
18 $\mathbf{u}_j \leftarrow \mathbf{u}_{j,l^*}$;
19 $l^* \leftarrow l_{j,l^*}^{\min}$;
20 end

hovering position can reach the current point, we will disregard it and retain its task completion time as ∞ to signify that it is unreachable. Then, in Line 16, we find the point above the last sensor that yields the minimum accumulated flight time. Finally, in Lines 17-20, we backtrack from the l^* -th point above sensor J to find the path that leads to the desired completion time.

It is worthwhile to note that, even though the DP principle is adopted to minimize the total completion time, the solution obtained by the above procedure may not necessarily be optimal since the set of feasible points in the previous position is restricted to the set $\mathcal{V}_{i,l}$, which contains the indices of points with sufficient energy to reach point $\mathbf{u}_{j,l}$. However, all but one path leading up to each point in \mathcal{U}_{i-1} are eliminated except for the path with minimum accumulated completion time. This may eliminate paths with more remaining energy, albeit with a slightly longer completion time. By doing so, the set of points that a point in \mathcal{U}_{i-1} can reach is limited. This sacrifice in optimality allows us to preserve the linear complexity of DP and, thus, obtain a low-complexity alternative to the HPO-SCA algorithm proposed in the previous section. But, it may affect the monotonicity required to prove the convergence of the MinTime-TCO algorithm. Hence, convergence is not theoretically guaranteed in this case. However, we see in the experiments (c.f., Fig. 6) that convergence can actually occur much faster than the original MinTime-TCO algorithm, significantly reducing the overall computation time.

Complexity Analysis: In the HPO-DP algorithm, we assume the maximum number of discretized points overall sensor nodes is given by $L \triangleq \max_{j=1,...,J} L_j$. The initialization in Line 1 requires complexity $\mathcal{O}(LJ)$. The computation of the variables associated with the first hovering position in Lines 2-5 requires complexity $\mathcal{O}(L)$. In lines 6 to 15, the complexity of the two **for** loops is $\mathcal{O}(LJ)$, and the calculation of $\mathcal{V}_{j,l}$ and $T_{j,l}$ is L. Thus, the complexity is $\mathcal{O}(L^2J)$. Finally, in Lines 16 to 20, the backtracking complexity is $\mathcal{O}(J)$. Combining all individual complexities, the overall complexity of HPO-DP is $\mathcal{O}(L^2J)$.

VI. OPTIMALITY PROOF OF THE MCRS ALGORITHM

In this section, we prove the optimality of the MCRS algorithm proposed in Section IV-A. We consider only the case where the data update time is negligible compared to the charging time, i.e., $\frac{D_j}{R_j(\mathbf{u}_j)} \ll \frac{e_j}{p_j^{\max}(\mathbf{u}_j)}$, for all j, and that the charging power is always greater than the hovering power consumption, i.e., $p_j^{\max}(\mathbf{u}_j) \ge P_0$. In this case, the optimization problem in (11) reduces to the following:

$$\min_{\{\mathbf{u}_{j}\}_{j=1}^{J},\{e_{j}\}_{j=1}^{J}} \sum_{j=1}^{J} \frac{e_{j}}{p_{j}^{\max}(\mathbf{u}_{j})} + \sum_{j=1}^{J-1} \frac{\|\mathbf{u}_{j} - \mathbf{u}_{j+1}\|}{V} \quad (49a)$$
subject to $E + \sum_{j'=1}^{j} e_{j'} \left(1 - \frac{P_{0}}{p_{j'}^{\max}(\mathbf{u}_{j'})}\right)$

$$- \sum_{j'=1}^{j+1} \frac{\|\mathbf{u}_{j'} - \mathbf{u}_{j'-1}\|}{V} P_{1} \ge 0, \,\forall j, \quad (49b)$$
 $E + \sum_{j'=1}^{j} e_{j'} \left(1 - \frac{P_{0}}{p_{j'}^{\max}(\mathbf{u}_{j'})}\right)$

$$- \sum_{j'=1}^{j} \frac{\|\mathbf{u}_{j'} - \mathbf{u}_{j'-1}\|}{V} P_{1} \le E, \,\forall j. \quad (49c)$$

Moreover, we assume that all the charging powers are distinct, i.e., $p_j^{\max}(\mathbf{u}_j) \neq p_{j'}^{\max}(\mathbf{u}_{j'})$, for all $j \neq j'$. In the general case where the upload time may not be negligible, we can simply treat the solution \mathbf{e}^j obtained at the end of each iteration as that obtained after exploiting the free charging opportunities up to that point, as described in the Free_Charge procedure. However, for ease of exposition, we do not treat this case explicitly in the following proof.

Let $\mathbf{e}^{j,\mathrm{opt}}_1 = (e_1^{j,\mathrm{opt}}, \ldots, e_J^{j,\mathrm{opt}})$ be the optimal charging solution that yields the minimum task completion time up to sensor j, i.e., the minimum time needed to collect data from sensors 1 to j. In this case, the remaining energy of the UAV upon arrival at position j should be equal to zero, i.e.,

$$E_{j}^{\text{in}}(\mathbf{e}^{j,\text{opt}}) \triangleq E + \sum_{j'=1}^{j-1} e_{j'}^{j,\text{opt}} \left(1 - \frac{P_{0}}{p_{j'}^{\max}(\mathbf{u}_{j'})}\right) - \sum_{j'=1}^{j} \frac{\|\mathbf{u}_{j'} - \mathbf{u}_{j'-1}\|}{V} P_{1} = 0$$
(50)

since the energy should be sufficient only to reach sensor j. Moreover, we have $e_j^{j,\text{opt}} = \cdots = e_J^{j,\text{opt}} = 0$ since the remaining sensors need not yet be visited. The following properties can be shown for $e^{j,\text{opt}}$.

Lemma 1: Let k_1 and k_2 (where $k_1 < k_2$) be the indices of two consecutive charging positions under solution $e^{j,\text{opt}}$ (i.e., $e_{k_1}^{j,\text{opt}} > 0$, $e_{k_2}^{j,\text{opt}} > 0$, and $e_k^{j,\text{opt}} = 0$, for all k such that $k_1 < k < k_2$). The following properties hold:

- i) If the UAV is not fully charged upon departure from position k_1 , i.e., $E_{k_1}^{\text{out}}(\mathbf{e}^{j,\text{opt}}) < E$, then k_2 must have a faster charging rate than k_1 , i.e., $p_{k_1}^{\max}(\mathbf{u}_{k_1}) < p_{k_2}^{\max}(\mathbf{u}_{k_2})$, and the remaining energy upon arrival at k_2 must be zero.
- ii) If the UAV is fully charged upon departure from k_1 , i.e., $E_k^{\text{out}}(\mathbf{e}^{j,\text{opt}}) = E$, and the remaining energy upon arrival at position k_2 is nonzero, then k_1 must have a faster charging rate than k_2 , i.e., $p_{k_1}^{\max}(\mathbf{u}_{k_1}) > p_{k_2}^{\max}(\mathbf{u}_{k_2})$.
- iii) The charging rates at positions k_1 and k_2 must be faster than that at any position in between, i.e., $p_{k_1}^{\max}(\mathbf{u}_{k_1}) > p_k^{\max}(\mathbf{u}_k)$ and $p_{k_2}^{\max}(\mathbf{u}_{k_2}) > p_k^{\max}(\mathbf{u}_k)$, for all k such that $k_1 < k < k_2$.
- iv) If k' is the last charging position under $\mathbf{e}^{j,\text{opt}}$ (i.e., $e_k^{j,\text{opt}} = 0$, for all k > k'), then $p_{k'}^{\max}(\mathbf{u}_{k'}) > p_k^{\max}(\mathbf{u}_k)$, for any k in between k' and j.

Proof: (i) First, we prove that $p_{k_1}^{\max}(\mathbf{u}_{k_1}) < p_{k_2}^{\max}(\mathbf{u}_{k_2})$ by contradiction. Suppose that $p_{k_1}^{\max}(\mathbf{u}_{k_1}) > p_{k_2}^{\max}(\mathbf{u}_{k_2})$. Since the battery capacity is not full upon departure at position k_1 , i.e., $E_{k_1}^{\text{out}}(\mathbf{e}^{j,\text{opt}}) < E$, the task completion time can be reduced (without violating the energy causality and battery capacity constraints in (49b) and (49c)) by increasing the charging energy $e_{k_1}^{j,\text{opt}}$ by Δe , where $\Delta e < \min\{E - E_{k_1}^{\text{out}}(\mathbf{e}^{j,\text{opt}}), e_{k_2}^{j,\text{opt}}\}$, and decreasing the charging energy $e_{k_2}^{j,\text{opt}}$ by the same amount Δe . This contradicts the fact that $\mathbf{e}^{j,\text{opt}}$ is optional.

The fact that the remaining energy upon arrival at k_2 is zero, i.e., $E_{k_2}^{\text{in}}(\mathbf{e}^{j,\text{opt}}) = 0$, can also be proven by contradiction. Suppose that $E_{k_2}^{\text{in}}(\mathbf{e}^{j,\text{opt}}) > 0$. Then, since $p_{k_1}^{\max}(\mathbf{u}_{k_1}) < \mathbf{e}^{\max}(\mathbf{u}_{k_1})$ $p_{k_2}^{\max}(\mathbf{u}_{k_2})$, a better solution can be obtained by reducing the charge amount at position k_1 by a small amount $\Delta e <$ $\min\{e_{k_1}^{j,\text{opt}}, E_{k_2}^{in}(\mathbf{e}^{j,\text{opt}}), E - E_{k_2}^{out}(\mathbf{e}^{j,\text{opt}})\}$ and by increasing the same amount at k_2 without violating the energy causality and batter capacity constraints. This again contradicts the fact that $e^{j,opt}$ is optimal. (ii) The result can again be proven by contradiction, similar to (i), and thus is omitted for brevity. (iii) Suppose that there exists k in between k_1 and k_2 such that $p_k^{\max}(\mathbf{u}_k) > k$ $p_{k_1}^{\max}(\mathbf{u}_{k_1})$. By the premise of the lemma, we know that the UAV is able to go from k_1 to k_2 under $e^{j,opt}$ without charging in between. This implies that the UAV can reach position k with nonzero remaining energy, i.e., $E_k^{\text{in}}(\mathbf{e}^{j,\text{opt}}) > 0$. In this case, we can obtain a better solution by reducing the charging energy at position k_1 by $\Delta e < \min\{E_k^{\text{in}}(\mathbf{e}^{j,\text{opt}}), E - E_k^{\text{out}}(\mathbf{e}^{j,\text{opt}})\}$ and increasing the same amount at position k without violating the energy causality and battery capacity constraints. Similarly, suppose that $p_k^{\max}(\mathbf{u}_k) > p_{k_2}^{\max}(\mathbf{u}_{k_2})$. In this case, we can again obtain a better solution by reducing the charging energy at k_2 by $\Delta e < \min\{e_{k_2}^{j, \text{opt}}, E_{k_2}^{\text{in}}(\mathbf{e}^{j, \text{opt}}), E - E_k^{\text{out}}(\mathbf{e}^{j, \text{opt}})\} \text{ and increas-}$ ing the same amount at position k. Hence, the result in (iii) is shown by contradiction. (iv) The proof is similar to that in (iii) and thus is omitted. \square

Given the properties of $e^{j,\text{opt}}$ shown in Lemma 1, we can then show in the following lemmas the relation between the solutions $e^{j,\text{opt}}$ and $e^{(j+1),\text{opt}}$. In particular, we show that $e^{(j+1),\text{opt}}$ can be obtained by charging the additional energy required to reach position j + 1 on top of the solution $e^{j,opt}$.

Lemma 2: Suppose that j^{full} is the last position under $e^{j,\text{opt}}$ at which the UAV is fully charged upon departure. Then, under solution $e^{(j+1),\text{opt}}$ (i.e., the optimal charging energy solution to reach position j + 1), the UAV must also be fully charged upon departure from the same position j^{full} .

Proof: We shall prove this result by contradiction. In particular, suppose that the UAV is not fully charged upon departure from location j^{full} under $e^{(j+1),\text{opt}}$, i.e., $E_{j^{\text{full}}}^{\text{out}}(e^{(j+1),\text{opt}}) < E$. Moreover, suppose that the non-zero charging positions after j^{full} are k_1, k_2, \ldots, k_L under $e^{j,\text{opt}}$, where $j^{\text{full}} < k_1 < k_2 < \cdots < k_L < j$. That is, $e_k^{j,\text{opt}} > 0$, for $k \in \{k_1, \ldots, k_L\}$, and is equal to 0, for all other k between j^{full} and j. Since j^{full} is the last time the UAV is fully charged, it follows from Lemma 1 (i), (iii), and (iv) that the charging rate at position k_L is higher than that at all positions k between j^{full} and j (i.e., $j^{\text{full}} < k < j$). Moreover, the charging rate at j^{full} is faster than at positions $j^{\text{full}} + 1$ to $k_1 - 1$, and the remaining energy upon arrival at positions k_2, \ldots, k_L , and j are zero.

If the remaining energy upon arrival at k_1 is zero under $e^{j,\text{opt}}$ (where the UAV is fully charged upon departure from j^{full}), then, since $E_{j^{\text{full}}}^{\text{out}}(\mathbf{e}^{(j+1),\text{opt}}) < E$, it is necessary to charge at some position from $j^{\text{full}} + 1$ to $k_1 - 1$ to gather enough energy to reach k_1 under $\mathbf{e}^{(j+1),\text{opt}}$. However, these positions have charging rates that are slower than j^{full} and, thus, a better solution can be obtained by charging at j^{full} and, thus, a better solution can be obtained by charging at j^{full} instead. This contradicts the fact that $\mathbf{e}^{(j+1),\text{opt}}$ is optimal. On the other hand, if $E_{k_1}^{\text{in}}(\mathbf{e}^{j,\text{opt}}) > 0$, then, by Lemma 1 (i) and (ii), we know that j^{full} has a faster charging rate than positions $j^{\text{full}} + 1$ to $k_2 - 1$. Since the energy upon arrival at k_2 is zero (i.e., $E_{k_2}^{\text{in}}(\mathbf{e}^{j,\text{opt}}) = 0$), by similar arguments, we can show that a better solution can be obtained since $E_{j^{\text{full}}}^{\text{out}}(\mathbf{e}^{(j+1),\text{opt}}) < E$.

Lemma 3: Suppose that j^{full} is the last position under $e^{j,\text{opt}}$ at which the UAV is fully charged. The charging solutions $e^{j,\text{opt}}$ and $e^{(j+1),\text{opt}}$ must be identical up to position j^{full} , i.e., $e_k^{(j+1),\text{opt}} = e_k^{j,\text{opt}}$, for $k = 1, \ldots, j^{\text{full}}$.

Proof: Let e be any solution that results in the UAV being fully charged at position j^{full} . In this case, we have $E_{j^{\text{full}}}^{\text{out}}(\mathbf{e}) = E$ or, equivalently,

$$\sum_{j'=1}^{j^{\text{full}}} e_{j'} - \sum_{j'=1}^{j^{\text{full}}} \frac{\|\mathbf{u}_{j'} - \mathbf{u}_{j'-1}\|}{V} P_1$$
$$- \sum_{j'=1}^{j^{\text{full}}} \max\left(\frac{D_{j'}}{R_{j'}(\mathbf{u}_{j'})}, \frac{e_{j'}}{p_{j'}^{\max}(\mathbf{u}_{j'})}\right) P_0 = 0 \quad (51)$$

By substituting the above into (11b), the constraints, for $j > j^{\text{full}}$, can be written as

$$E + \sum_{j'=j^{\text{full}}+1}^{j} e_{j'} - \sum_{j'=j^{\text{full}}+1}^{j+1} \frac{\|\mathbf{u}_{j'} - \mathbf{u}_{j'-1}\|}{V} P_1$$
$$- \sum_{j'=j^{\text{full}}+1}^{j} \max\left(\frac{D_{j'}}{R_{j'}(\mathbf{u}_{j'})}, \frac{e_{j'}}{p_{j'}^{\max}(\mathbf{u}_{j'})}\right) P_0 \ge 0.$$
(52)

Notice that the above constraint is no longer dependent on the solutions before j^{full} . Therefore, the constraints in (11b) become decoupled for $j \leq j^{\text{full}}$ and $j > j^{\text{full}}$. The same can be shown for the constraints in (11c). Hence, given that the UAV is fully charged at position j^{full} , the optimization problem in (11) can be solved separately for $j \leq j^{\text{full}}$ and $j > j^{\text{full}}$ since the objective function is additive.

By Lemma 1, we know that the UAV must be fully charged at position j^{full} under both $e^{j,\text{opt}}$ and $e^{(j+1),\text{opt}}$. This implies that both $e^{j,\text{opt}}$ and $e^{(j+1),\text{opt}}$ can be solved separately for $j \leq j^{\text{full}}$ and $j > j^{\text{full}}$. Hence, the solutions obtained by both $e^{j, \text{opt}}$ and $e^{(j+1),opt}$ up to j^{full} must be identical.

Using the above lemmas, we can then prove the optimality of the MCRS algorithm in terms of minimizing the overall task completion time. The results are summarized in the following theorem.

Theorem 2: The MCRS algorithm yields the optimal solution to the charge time minimization problem in (12).

Proof: The theorem is proved by induction. First, for j = 2(i.e., the base case), the MCRS algorithm charges at position 1 the minimum energy required to reach position 2. Since position 1 is the only possible charging position before position 2, the MCRS algorithm yields the optimal charging solution for reaching position 2, i.e., $e^{2,opt}$.

For the inductive step, suppose that the MCRS algorithm yields the optimal solution for reaching position j, i.e., $\mathbf{e}^{j,\mathrm{MCRS}} = \mathbf{e}^{j,\mathrm{opt}}$, and let j^{full} be the last position before jat which the UAV is fully charged. The MCRS algorithm builds upon the solution $e^{j,opt}$ by additionally charging, in each iteration, at the position with the fastest charging rate after the last fully charged position. The process is repeated until the minimum required energy to reach j + 1 is acquired. Hence, the solution remains unchanged before position j^{full} . By Lemma 3, we know that the optimal charging solution $e^{(j+1),opt}$ must yield the same solution as $e^{j,opt}$ up to position j^{full} , i.e., $e_k^{(j+1),\text{opt}} = e_k^{j,\text{opt}}$, for $k = 1, \dots, j^{\text{full}}$. Hence, the MCRS solution $e^{(j+1),\text{MCRS}}$ is optimal up to the j^{full} -th entry, i.e., $e_k^{(j+1),\text{MCRS}} = e_k^{j,\text{opt}} = e_k^{(j+1),\text{opt}}$, for $k = 1, \dots, j^{\text{full}}$. Hence, it remains to be shown that the charging behavior of the MCRS algorithm is optimal at positions after j^{full} .

Let k_1, k_2, \ldots, k_L be the non-zero charging positions between j^{full} and j under $e^{j,\text{opt}}$. That is, $e_k^{j,\text{opt}} > 0$, for $k \in \mathbb{R}$ $\{k_1, \ldots, k_L\}$, and is equal to 0, for all other k between j^{full} and j. Since j^{full} is the last time the UAV is fully charged, it follows from Lemma 1 (i), (iii), and (iv) that the charging rate at position k_L is faster than that at all positions k such that $j^{\text{full}} < k < j$. However, it is unclear whether the charging rate is higher at position j or position k_L . In the following, we consider separately the case where the charging rate at position *j* is higher than that at position k_L and the case where it is lower.

Suppose that the charging rate at position j is higher than that at position k_L . Then, the charging rate at position j must be the fastest among all positions after j^{full} . In this case, the MCRS algorithm builds upon the solution $e^{j,opt}$ by charging at position j the minimum amount required to reach position j + 1, i.e., $\Delta E_{j+1}^{\text{req}}(\mathbf{e}^{j,\text{MCRS}}) = \frac{\|\mathbf{u}_{j+1}-\mathbf{u}_{j}\|}{V}P_{1}$, while leaving the solution unchanged up to position j - 1. Note that charging the amount $\Delta E_{j+1}^{\mathrm{req}}(\mathbf{e}^{j,\mathrm{MCRS}})$ at any position other than j would increase the task completion time since j has the highest charging rate. Hence, the MCRS solution to reach position j + 1 is optimal.

On the other hand, suppose that the charging rate at position k_L is higher than that at position j (and thus all positions in between). In this case, k_L is the fastest charging position after j^{full} . Suppose that the minimum energy required to reach j+1 is less than the battery capacity at position k_L , i.e., $\Delta E_{j+1}^{\text{req}}(\mathbf{e}^{j,\text{MCRS}}) < E - E_{k_L}^{\text{out}}(\mathbf{e}^{j,\text{opt}}).$ In this case, the MCRS algorithm chooses to charge the energy required at position k_L while leaving the solutions at other positions unchanged. This solution is optimal since charging at any other position would yield a longer completion time.

Now, suppose that $\Delta E_{j+1}^{\text{req}}(\mathbf{e}^{j,\text{MCRS}}) > E - E_{k_L}^{\text{out}}(\mathbf{e}^{j,\text{opt}})$. In this case, the MCRS algorithm builds upon the solution $e^{j,opt}$ by fully charging the battery in turn at positions $k_L, k_{L+1}, k_{L+2}, \ldots$ until the minimum energy required to reach j + 1 is acquired. The positions k_{L+m} , for m = 1, 2, ..., are chosen such that $p_{k_{L+m}}^{\max}(\mathbf{u}_{k_{L+m}}) > p_{k'}^{\max}(\mathbf{u}_{k'}), \text{ for all } k' > k_{L+m-1}.$ Suppose the last charging position is k_{L+M} . In this case, the UAV must be fully charged upon departure from each of these positions unless m = M.

Suppose the UAV is not fully charged at position k_L under the solution $e^{(j+1),opt}$. This implies that a portion of $\Delta E_{j+1}^{\mathrm{req}}(\mathbf{e}^{j,\mathrm{MCRS}})$ must be acquired by charging at some point k'after k_L . In this case, a better solution can be obtained by increasing the charging energy at k_L while reducing the same portion at k' since k_L has a higher charging rate than k'. Furthermore, since the UAV is fully charged at k_L , by the same argument as the proof of Lemma 3, the charging solution $e^{(j+1), opt}$ must remain the same as $e^{j,opt}$ before position k_L . Hence, fully charging the battery at position k_L and not charging anywhere else between j^{full} and k_L , as done in the MCRS algorithm, is optimal.

Similarly, given that k_L is fully charged, the optimal solution must also be fully charged at k_{L+1} since k_{L+1} has the fastest charging rate for all positions after k_L . If k_{L+1} is not fully charged, then the missing energy must be charged at some position k' after k_L . In this case, a better solution can be obtained by increasing the charging energy at k_{L+1} while reducing the same amount at k'. In fact, no energy should be charged at any position in between k_L and k_{L+1} since charging at any of these positions implies that less energy can be charged at k_{L+1} and thus a longer charging time is required. Following similar arguments, we can show that the optimal solution must yield fully charged batteries at positions k_L, k_{L+1}, \ldots , until the minimum required energy $\Delta E_{j+1}^{\text{req}}(\mathbf{e}^{j,\text{MCRS}})$ is acquired, which is consistent with the MCRS algorithm.

VII. SIMULATION RESULTS

In this section, we provide numerical simulations to demonstrate the effectiveness of our proposed algorithms. In these experiments, we randomly deploy J = 70 sensors according to a uniform distribution across a $2000 \times 2000 \text{ m}^2$ square area. I =3 HAPs are placed at an altitude of $a_{i,3} = 1000$ m. The horizontal coordinates of the HAPs, i.e., $a_{i,1}$ and $a_{i,2}$, are chosen as the centroids of the k-means clustering [35] of the J sensor nodes.

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Description	Value
Number of Sensors	J = 70
Number of HAPs	I = 3
Network area	$2000 \times 2000 \text{ m}^2$
UAV velocity [37]	V = 5 m/s
Battery capacity [37]	E = 100 kJ
Minimum and maximum UAV altitudes	$H_{\min} = 100 \text{ m}$ $H_{\max} = 500 \text{ m}$
Transmission power of sensors [38]	$P^{\mathrm{tx}} = 0.1 \mathrm{mW}$
Transmission bandwidth [38]	B = 20 MHz
Noise power spectral density	$N_0 = -123.83 \text{ dBm/Hz}$
Constants in LoS probability model [26]	$C_1 = 0.136$ $C_2 = 11.95$
Reference channel gains [26]	$ \rho_0^{\text{LoS}} = 10.38 $ $ \rho_0^{\text{NLoS}} = 14.54 $
Path loss exponents [26]	$\begin{array}{l} \alpha^{\rm LoS} = 2.09\\ \alpha^{\rm NLoS} = 3.75 \end{array}$
Propulsion powers for 0 and $V m/s$ [29]	$P_0 = 110 \text{ W}$ $P_1 = 130 \text{ W}$
Laser charging power	$P^L = 750 \mathrm{W}$
Energy harvesting efficiency [20]	$\eta = 0.004$
Attenuation coefficient [20]	$\gamma = 10^{-6}$
Initial beam size [20]	$\zeta = 0.1 \text{ m}$
Angular spread of beam [20]	$\phi = 3.4 \times 10^{-5}$

TABLE I SIMULATION PARAMETERS

The laser charging power is $P^L = 750$ W. We adopt a visiting order that is obtained by solving the Traveling Salesman Problem over the sensors using a 2-approximation ratio algorithm [36]. The visiting order is fixed for all comparison schemes. Other visiting orders can certainly be used in our experiments as well but is not the focus of our comparisons. The data sizes $\{D_i\}_{i=1}^J$ are i.i.d. uniformly distributed between 250 and 350Mbps. We set the UAV velocity as V = 5 m/s and the battery capacity as E = 100 kJ, which is in the order of DJI-manufactured UAV systems [37]. The minimum and maximum UAV altitudes are set as $H_{\min} = 100$ and $H_{\max} = 500$ meters, respectively. Notice that, following [23], we set the altitude of HAPs in the order of kilometers and the altitude of UAVs in the order of hundreds of meters. The sensor nodes use a transmission power of $P^{tx} = 0.1$ mW and a bandwidth of B = 20 MHz to upload data [38]. Moreover, the parameters pertaining to the LoS probability model are inherited from [26]. In particular, the constants C_1 and C_2 related to the LoS probability model are set as 0.136 and 11.95, respectively, the reference channel gains for the LoS and NLoS channels are $\rho_0^{\text{LoS}} = 10.38$ and $\rho_0^{\text{NLoS}} = 14.54$, and the path loss exponents are $\alpha^{\text{LoS}} = 2.09$ and $\alpha^{\text{NLoS}} = 3.75$ [26]. The noise power spectrum density is $N_0 = -123.83$ dBm/Hz. The UAV's propulsion power consumption for velocities 0 and V m/s are set as $P_0 = 110$ W and $P_1 = 130$ W, respectively, which are computed based on the power consumption model and corresponding parameters in [29]. The harvesting efficiency of the UAV is $\eta = 0.004$, the attenuation coefficient of the medium is $\gamma = 10^{-6}$, the size of the initial laser beam is $\zeta = 0.1$ m, and the angular spread of the beam is $\phi = 3.4 \times 10^{-5}$ [20]. We summarize the list of parameters in Table I.

In Figs. 3–5, we compare different hovering position optimization strategies, namely, the proposed HPO-SCA and HPO-DP algorithms, the Bat algorithm [39], and the min-max approach (MinMax). The Bat algorithm is a nature-inspired optimization method that was adopted in [39] for UAV position



Fig. 3. Comparison of task completion times with respect to laser power for different hovering position optimization strategies.

optimization. The MinMax approach requires the UAV to hover directly above the corresponding sensor nodes for both data upload and energy charging, with heights chosen to minimize the maximum time between data upload and charging. The MCRS algorithm is adopted as the energy charging solution in all cases. More specifically, to solve our constrained hovering position optimization problem with the Bat algorithm, we follow the approach given in [40] by considering a modified objective function $f({\mathbf{u}}_j)_{j=1}^J + \sum_{j=1}^J C(\min(g_j({\mathbf{u}}_j), 0))^2)$, where $f({\mathbf{u}_j}_{j=1}^J)$ is the original objective function in (18a) (or equivalently (20a)), $g_j(\mathbf{u}_j)$ is the left-hand-side of the constraint in (20b), and C is a large constant serving as the violation penalty (which is chosen as C = 100 in our experiments). In the Bat algorithm, we set the minimum and maximum frequencies as $f_{\min} = 0$ and $f_{\max} = 5$, the loudness as A = 0.75, pulse rate as r = 0.5, and the number of bats equal to 1000. The location of each bat can be randomly perturbed by an amount that is Gaussian with standard deviation 0.01. For the HPO-SCA algorithm, the initial hovering positions are chosen as points directly above the sensors at altitude 500 m.

Specifically, in Fig. 3, we show the task completion times with respect to laser power P^{L} . Our proposed schemes significantly outperform the MinMax approach, which requires UAVs to hover directly above the sensors when collecting data. This may require UAVs to expend more power to reach these locations when they could otherwise have received data at a distance from the sensor to reduce the flight time and obtain a better charging position. Our proposed schemes also outperform the Bat algorithm, which adopts a local search based on a few initial solutions, but can be easily trapped inside locally optimal solutions. Moreover, we can see that the HPO-DP algorithm performs close to the HPO-SCA algorithm while requiring significantly lower computational complexity (see later in Table II and Fig. 6).

In Fig. 4, we show the task completion times with respect to the number of sensors J. We can again see that our proposed HPO-SCA and HPO-DP algorithms significantly outperform both the MinMax and Bat algorithms. More interestingly, even



Fig. 4. Comparison of task completion times with respect to the number of sensors for different hovering position optimization strategies.



Fig. 5. Comparison of task completion times with respect to the network area for different hovering position optimization strategies.

though the completion time increases with the number of sensors in all cases, the impact on the HPO-SCA and HPO-DP schemes is less than that of the MinMax and Bat algorithms. This is because, as the number of sensors increases, the UAV's flight distance does not need to increase significantly since it can collect data from multiple sensors at approximately the same location. However, this is not the case for the MinMax strategy, which must visit locations directly above the sensors for data collection. The increase in dimension as the number of sensors increases also limits the effectiveness of the local search in the Bat algorithm.

In Fig. 5, we show the task completion times with respect to the network area. Increasing the network area causes the sensors to be located farther apart and thus the UAVs need to fly longer distances for data collection. Therefore, the task completion time increases with the network area in all cases. In fact, the advantage of the proposed HPO-SCA and HPO-DP algorithms becomes more significant as the network area increases since the MinMax

TABLE II EXECUTION TIME FOR VARYING NUMBER OF SENSOR NODES



Fig. 6. Convergence of HPO-SCA and HPO-DP algorithms.

approach needs to visit the locations directly above the sensors, which are now located farther apart. Although the Bat algorithm does not outperform our proposed algorithm, its task completion time does not increase as rapidly as that of MinMax.

In previous experiments, we observed that the HPO-DP algorithm performs close to HPO-SCA while requiring significantly shorter computation time. This is shown explicitly in Table II, where we can see that the average execution time of HPO-DP for J = 70 sensors is 40.73 seconds as opposed to 767.09 seconds for HPO-SCA. The difference further increases with the number of sensors. In Fig. 6, we show the number of iterations required for the two proposed algorithms. We can see that the HPO-DP algorithm requires only around 2 iterations to converge, whereas the HPO-SCA algorithm requires about 7 iterations. However, HPO-SCA can converge to a better solution since it gradually updates the hover positions in each iteration through successive approximation as opposed to directly moving to the best hover positions in HPO-DP. Since HPO-DP can achieve similar performance with much lower complexity, we focus on the use of HPO-DP in the following comparisons between different energy charging strategies.

By adopting the HPO-DP algorithm for hovering position optimization, we then compare with four baseline energy charging strategies in Figs. 7-9. That is,

- *Fully Charge (FC):* In this method, the UAV is fully charged upon departure from each position.
- *Greedy Charge (GC):* The UAV is charged at each hover position the minimum amount needed to reach the next position.
- *Greedy Fully Charge (GFC):* Instead of charging the minimum amount, the UAV is fully charged whenever the remaining energy is insufficient to reach the next position.



Fig. 7. Comparison of task completion times with respect to laser power for different charging strategies.



Fig. 8. Comparison of task completion times with respect to the number of sensor nodes for different charging strategies.



Fig. 9. Comparison of task completion times with respect to the network area for different charging strategies.



Fig. 10. Details of the time composition with respect to the number of sensor nodes for different charging strategies.

• Drones Traveling Algorithm (DTA) [23]: A laser-powered UAV data-gathering scheme proposed in [23], where the laser-charging locations are fixed directly below the HAPs and, thus, the UAV must travel to these designated locations for charging.

In Fig. 7, we show the task completion time with respect to laser power for different energy charging strategies, namely, FC, GC, GFC, DTA, and the proposed MCRS. We can observe that MCRS outperforms other baseline schemes regardless of the laser power. This is not surprising since MCRS was shown to be optimal in terms of minimizing the task completion time. The advantage is most evident when the laser power is small since MCRS is able to consistently find positions with the maximum charge rate for charging. Notice that DTA [23] performs poorly due to the need to reach designated charging positions and to perform charging and data collection separately. The charging altitude in DTA is chosen as 400 m, which is optimized by line search.

In Fig. 8, we show the task completion time versus the number of sensor nodes. The proposed MCRS algorithm again outperforms all other baseline charging strategies. However, even though both the total data and flight time increases with the number of sensors, the task completion time does not increase as rapidly. This is because an increase in the number of sensor nodes provides the UAV with more opportunities to charge while collecting data, avoiding additional delays due to charging.

In Fig. 9, we show the task completion time with respect to the area size of the sensor field. We can see that the MCRS algorithm again achieves the minimum task completion time among all charging strategies. The advantage of MCRS over other baseline strategies becomes more pronounced as the area size increases. This is because, as the area size increases, the distance between hovering positions will also increase, causing the UAV to consume more energy and, thus, gain more by using a more efficient charging strategy.

In Fig. 10, we analyze the composition of the task completion times for different energy charging strategies. In particular, the

Charge/Upload Time 10000 Charge Time MCRS+MinMax 9000 Task Completion Time (s) Upload Time MCRS-Flight Time MCRS+MinMax 8000 CRS+MinMax 7000 MCRS+MinMax MCRS+Bat [36 MCRS+Bat [36 6000 MCRS+Bat [36 MCRS+HPO-DF MCRS+HPO-DF MCRS+HPO-SC. MCRS+HPO-SC MCRS+HPO-SC MCRS+HPO MCRS+Bat MCRS+HPO-SC MCRS+HPO-5000 ACRS+HPO MCRS+HPO MCRS-4000 +Bat [36 3000 2000 1000 0 50 60 70 80 90 Number of Sensor Nodes J

Details of the time composition with respect to the number of sensor Fig. 11. nodes for different hovering position optimization strategies.

task completion time is divided into flight time and hovering time, where the latter is further categorized into charge time, upload time, and charge/upload time. Charge time indicates the time used for charging only, without collecting data from the sensors. Upload time is the time used for data upload only, without carrying out charging simultaneously. Charge/upload time represents the time used for simultaneous charging and data upload. We observe that the flight time and total upload time (including both upload and charge/upload times) are roughly the same for different charging strategies. The variations in task completion times mainly come from the charging time that is needed beyond the free charging that is received during data upload. The ability of MCRS to fully exploit this advantage leads to the minimum task completion time among all charging strategies.

In Fig. 11, we compare the composition of task completion times for different hovering position optimization strategies. We can see that all schemes well utilize simultaneous charging during data upload since MCRS is considered in all cases. Even though MinMax aims to minimize the maximum charging and upload times at each hovering position, it leads to longer flight distance between hovering positions and, thus, more energy consumption.

VIII. CONCLUSION

In this work, we examined the trajectory design and energy charging strategy for a data-gathering UAV utilizing laser charging from multiple HAPs to replenish the UAV's battery. We proposed the MinTime-TCO algorithm that considers the interplay between data upload and energy charging efficiency to minimize the overall task completion time. The proposed MinTime-TCO algorithm employs a BCD approach, where the UAVs' hovering positions and charging energies are optimized in turn until convergence. Given the hovering positions, we proposed the MCRS algorithm to optimize the charging energies at the different hovering positions. An optimality proof of the MCRS algorithm was also provided. Then, given the energy charging amounts, the hovering positions were optimized by the

HPO-SCA algorithm, which utilizes SCA to address the nonconvexity of the optimization problem. An alternative HPO-DP algorithm was also proposed to reduce the complexity at little performance loss. Finally, numerical simulations were provided to validate the efficacy of the proposed algorithms against several baseline approaches.

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