Energy Efficiency Modeling and Analysis in Wireless Sensor Networks[*]

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Abstract

We propose here an extended linear feedback model, taking the binary exponential backoff mechanism adopted in IEEE 802.15.4 CSMA/CA and analyze the energy consumption issues of the one hop sensor nodes. Numerical results show that the energy consumption in Wireless Sensor Networks (WSNs) can be reduced by applying CSMA protocol with fixed contention window size. Besides, an optimal contention window size can achieve the reasonable successful probability of the packet transmission without extra wastage of the battery power.

1 Introduction

The IEEE 802.15.4 specification [1] now provides a standardized base set of solutions for low-rate wireless personal area networks (LR-WPANs), which can be easily used for implementing WSNs. Due to the power constraints characteristics of the sensors, it is vital to enhance the longevity of the nodes in the network to prolong the network lifetime. Though, several studies propose for the WSNs to be long-lived, analytical methods are rarely proposed along this direction. It is observed that most of the works are focused on the analytical model for the IEEE 802.11 WLAN (Wireless Local Area Network) performance evaluation and have studied how to improve the system throughput with their analytical models. In [2], Bianchi provides a famous framework for studying the saturation throughput of the Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) algorithm adopted for IEEE 802.11 Distributed Coordination Function (DCF). In [3], the authors extend the model introduced in [2] to study the system throughput without the saturation assumption. A Markovian state dependent continuous time, single server queue (SSQ) is used to represent the process of the idle stations that become active. The authors in [4] propose an analytical model for the slow contention window (CW) decrease scheme for the IEEE 802.11 MAC protocol to prevent from plenty collisions introduced by the overloaded traffic. On the other hand, the authors in [5] modify the linear feedback model in [6], for the IEEE 802.11 CSMA/CA and declare that the system throughput can be improved through the optimal contention window size that they have found. From the lately published studies [7, 8], we notice that researchers are diverting their attention from WLANs (IEEE 802.11) to LR-WPANs (IEEE 802.15.4). Motivated with the importance of the energy conservation in WSNs, we develop here a rigorous analytical model based on the linear feedback model [6] for analyzing and computing the energy efficiency of wireless sensor networks using CSMA/CA as the basic channel access mechanism.

The rest of the paper is organized as follows. Section 2 introduces the system model of our analysis. Section 3 describes the theoretical model that we have designed for evaluating the expected energy consumption in wireless sensor networks. Performance analysis and numerical results of our energy model are shown in Section 4. Concluding remarks are made in Section 5 of the paper.

2 System Model

2.1 The Extended Linear Feedback Model

Let us consider a homogeneous WSNs that consists of N number of nodes without hidden terminals, where nodes may be in the *thinking* or *backlogged* state, alternatively. Nodes in thinking state may generate new packets with probability g, whereas it remains in backlogged state if the medium sensed by it is busy due to the data transmission by other nodes of the network or due to collision of its packets with others. As shown in Fig 1, in the extended linear feedback model, $L+1$ number of backlog states are considered, where L is the retry limit which is application oriented or set as default value as per IEEE 802.15.4 standard. The retransmission probabilities introduced by the binary exponential backoff mechanism adopted by the IEEE 802.15.4 CSMA, is doubled with each retransmissions.

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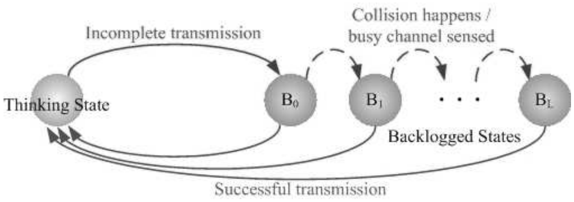

Figure 1: The extended linear feedback model.

Let, W_0 be the initial size of the contention window. As the contention window in the extended linear feedback model is doubled for each retransmission, the contention window of the r-th retransmission is defined as $W_r = W_0 \times 2^r$. Since the contention window size in our model is W_r, the rescheduling delay of a backlogged packet will then be uniformly distributed with a mean of $W_r/2$ slots, where r is the number of retransmissions of a backlogged packet. So each backlogged user senses the channel in the current slot of r-th retransmission with a probability $v_r = 2/W_r$. It is to be noted that the backlogged nodes can perform retransmissions unless the channel is assessed to be idle. Once the transmission attempt is either completed successfully or rejected owing to the retransmission limit, a backlogged device can immediately switch back to the thinking state.

2.2 A Discrete-Time Markov Chain Model

As per the Extended Linear Feedback Model, there are $L+1$ number of backlogged states present in the system. Let B_0, B_1,...,B_L represent those backlogged states. Let, i_0, i_1,...,i_L are the number of backlogged nodes present within the backlogged states B_0, B_1,...,B_L respectively and X_t denotes the total number of backlogged nodes present within all those backlogged states B_r, $\forall\ r \in \{0, 1 \ldots, L\}$. So $X_t = \sum_{r=0}^{L}(i_r)$, for all $t \in [t, t+I]$, where I is length of an idle slot. So the total number of backlogged nodes X_t, among all the backlogged states may range from 0 to N and the $L+1$ number of backlogged states can be represented in a discrete-time Markov chain as shown in the Fig 2. The transition from state i to state j $(i \leq j)$ means that there are some thinking terminals entering to the backlogged state. Similarly, the transition from state $i+1$ to state i represents that there is a successful packet transmission.

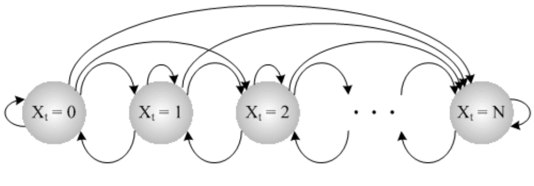

Figure 2: A discrete-time Markov chain model for system status.

Let, the communication channel consists of a sequence of regeneration cycles composed of idle and busy periods. The embedded slots are defined to be the first slot of each idle period. As shown in Fig 3, length of an idle period is I slots and length of the busy period is T slots. The only one slot Q', after end of each transmission, accounts for the propagation delay. So, each cycle of the communication channel occupies $I+T$ number of slots. The busy period has length T, if the transmission is successful, otherwise it has C slots if it is unsuccessful due to collision. By definition, no sensor node is ready for transmission during the interval $[T+1, T+I-1]$; however at least one node becomes ready in the last slot of the idle period i.e. at the $(T+I)$-th slot. Nodes, those who become ready at the $(T+I)$-th slot will sense the channel idle and will transmit at the beginning of the $(T+I+1)$-th slot.

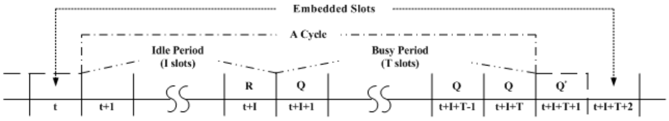

Figure 3: Embedded slots and communication cycle of Markov Chain.

Considering the thinking nodes generate new packets with probability g and the backlogged nodes retransmit the buffered packets with probability v_m, the conditional probability that at least one node is ready when the system status $X_{t+I} = i$ is given by

$$\begin{aligned} & Pr\,[\text{at least one node is ready} | X_{t+I} = i] \\ & \quad = 1 - \left(\prod_{m=0}^{L} (1 - v_m)^{i_m} \right) (1 - g)^{N-i} \end{aligned} \tag{1}$$

We denote R as the state transition matrix for the last idle slot $t+I$, and Q for all remaining time slots of the busy periods from the slot $t+I+1$ to $t+I+T$ for the successful transmission and $t+I+1$ to $t+I+C$ for the unsuccessful transmission due to collision in the channel, where C represents the number of slots due to collision. We specify the transition probability matrix R= $S+F$, where the $(i,\ k)$-th element of S and F are defined as

$$\begin{aligned} (s_{ik}) &= Pr[X_{t+I+1} = k \\ & \text{and transmission is successful} | X_{t+I} = i] \end{aligned} \tag{2}$$

and

$$\begin{aligned} (f_{ik}) &= Pr[X_{t+I+1} = k \\ & \text{and transmission is failed} | X_{t+I} = i] \end{aligned} \tag{3}$$

For any $t \in [t+I+2, t+I+T]$, we define the one-step transition probability matrix Q

$$q_{ik} = Pr[X_{t+1} = k | X_t = i] \tag{4}$$

If the transmission is successful, the busy period's length is T slots and if it is unsuccessful, its length is C slots. So the transmission matrix P, is expressed as

$$P = SQ^T J + FQ^C \tag{5}$$

where S, F, and Q are defined as follows

$$s_{ik} = \begin{cases} 0,\ k < i \\ \left[\frac{\sum_{h=0}^{L}\left(i_h v_h (1-v_h)^{i_h-1}\prod_{m=0,m\neq h}^{L}(1-v_m)^{im}\right)}{1-\left(\prod_{m=0}^{L}(1-v_m)^{im}\right)(1-g)^{N-i}}\right] \\ \times(1-g)^{N-i},\ k=i \\ \frac{\left(\prod_{m=0}^{L}(1-v_m)^{im}\right)(N-i)g(1-g)^{N-i-1}}{1-\left(\prod_{m=0}^{L}(1-v_m)^{im}\right)(1-g)^{N-i}},\ k = i+1 \\ 0,\ k > i+1 \end{cases} \tag{6}$$

$$f_{ik} = \begin{cases} 0, \text{ for } k < i \\ \left[\frac{1-\left(\prod_{m=0}^{L}(1-v_m)^{im}\right)}{1-\left(\prod_{m=0}^{L}(1-v_m)^{im}\right)(1-g)^{N-i}} - \right. \\ \left. \frac{\sum_{h=0}^{L}\left(i_h v_h (1-v_h)^{i_h-1}\prod_{m=0,m\neq h}^{L}(1-v_m)^{im}\right)}{1-\left(\prod_{m=0}^{L}(1-v_m)^{im}\right)(1-g)^{N-i}}\right] \\ \times(1-g)^{N-i}, \text{ for } k=i \\ \frac{\left(1-\left(\prod_{m=0}^{L}(1-v_m)^{im}\right)\right)(N-i)g(1-g)^{N-i-1}}{1-\left(\prod_{m=0}^{L}(1-v_m)^{im}\right)(1-g)^{N-i}} \\ , \text{ for } k = i+1 \\ \frac{\begin{pmatrix} N-i \\ k-i \end{pmatrix} g^{k-i}(1-g)^{N-k}}{1-\left(\prod_{m=0}^{L}(1-v_m)^{im}\right)(1-g)^{N-i}}, \text{ for } k > i+1 \end{cases} \tag{7}$$

and

$$q_{ik} = \begin{cases} 0 & , k < i \\ \begin{pmatrix} N-i \\ k-i \end{pmatrix} g^{k-i}(1-g)^{N-k} & , k \geq i \end{cases} \tag{8}$$

where J represents the fact that a successful transmission decreases the backlog by 1. So its (i, k)-th entry is defined as follows

$$j_{ik} = \begin{cases} 1 & , k = i-1 \\ 0 & , otherwise \end{cases} \tag{9}$$

Let $\Pi = (\pi_0, \pi_1, \ldots, \pi_N)$ denotes the stationary probability distribution of X_t at the embedded points, where π_i represents the probability that there are i nodes in the backlogged state at the current slot. Then, we have $\Pi = \Pi P$, which can be obtained from the recursive solution of $\Pi = \Pi P$.

3 Energy Consumption Analysis

The analysis is based on the extended linear feedback model and the state transition probability, presented in the previous sections.

3.1 Energy Consumption Model

Let, $\bar{P}_s(r)$: be the expected successful probability of the r-th retransmission of transmission attempts, for $r \in \{1, \ldots, L\}$. $\bar{P}_s(0)$: be the expected successful probability of the first transmission. $\epsilon_s(r)$: be the total energy consumption of the successful transmission attempt with r number of retransmissions. $\epsilon_f(r)$: be the total energy consumption of the failed transmission attempts with r number of retransmissions. Then the expected energy consumption for any transmission attempts, due to L-retransmission attempts can be estimated as follows:

$$\begin{aligned} \bar{\epsilon} &= \bar{P}_s(0)\epsilon_s(0) + \sum_{r=1}^{L}\left(\prod_{j=0}^{r-1}\left[1-\bar{P}_s(j)\right]\right)\bar{P}_s(r)\epsilon_s(r) \\ &+ \left(\prod_{r=0}^{L}\left[1-\bar{P}_s(r)\right]\right)\epsilon_f(L) \end{aligned} \tag{10}$$

Since the system status varies with time, we have the expected successful probability of the r-th retransmission of the transmission attempts as follows:

$$\bar{P}_s(r) = \sum_{i=0}^{N}\pi_i P_s(r,i) \tag{11}$$

where π_i is the probability that the system status X_t equals to i. $P_s(r,i)$ is the successful probability of the r-th retransmission of the transmission attempt while there are i nodes in the backlogged state. The successful probability $P_s(r,i)$ is given as follows:

$$P_s(r,i) = \begin{cases} \left[\frac{(N-i)g(1-g)^{N-i-1}\left(\prod_{m=0}^{L}(1-v_m)^{im}\right)}{1-\left(\prod_{m=0}^{L}(1-v_m)^{im}\right)(1-g)^{N-i}}\right] + \\ \left[\frac{\left(i_r v_r (1-v_r)^{i_r-1}\prod_{m=0,m\neq r}^{L}(1-v_m)^{im}\right)(1-g)^{N-i}}{1-\left(\prod_{m=0}^{L}(1-v_m)^{im}\right)(1-g)^{N-i}}\right] \\ , \text{ for } r = 0 \\ \frac{\left(\prod_{m=0}^{L} i_r v_r (1-v_r)^{i_r-1}\prod_{m=0,m\neq r}^{L}(1-v_m^{im}\right)(1-g)^{N-i}}{1-\left(\prod_{m=0}^{L}(1-v_m)^{im}\right)(1-g)^{N-i}} \\ , \text{ for } 0 < r \leq L \end{cases} \tag{12}$$

3.2 Energy Consumption Analysis

In this section we analyze the energy consumption due to failure and success of each transmission due to retransmission attempts. Generalizing for any retry limit r, the total energy consumption is given by

$$\epsilon_s(r) = r \cdot \epsilon_f + \epsilon_s + \sum_{k=0}^{r}\bar{\epsilon}_{backoff}(k) \tag{13}$$

$$\epsilon_f(r) = (r+1) \cdot \epsilon_f + \sum_{k=0}^{r}\bar{\epsilon}_{backoff}(k) \tag{14}$$

where,

$\epsilon_s = \epsilon_{cca} + \epsilon_{data} + \epsilon_{ack}$

$\epsilon_f = \epsilon_{cca} + \epsilon_{data} + \epsilon_{ack\_tout}$

ϵ_s is the energy consumption of a successful transmission excluding the energy consumption during the backoff period. The value of ϵ_s comprises the energy spent for performing CCA (ϵ_{cca}), energy spent for transmitting data frame (ϵ_{data}), the energy spent in the interframe space time (ϵ_{ifs}), and the energy spent for receiving the acknowledgement message (ϵ_{ack}). On the contrary, ϵ_f is the energy consumption of a failed transmission excluding the energy consumption during backoff period. The value of ϵ_f considers the energy spent for performing CCA (ϵ_{cca}), energy spent for transmitting data frame (ϵ_{data}), and the energy spent for waiting the acknowledgement timeout (ϵ_{ack_tout}). Let P_{tx_mode} and P_{rx_mode} denote the power consumption in the transmit and receive mode, respectively.

To be more accurate, the analysis of the expected energy spent during the backoff period ($\bar{\epsilon}_{backoff}(r)$) in the r-th retransmission of all transmission attempts is divided into two parts: the energy consumption while the backoff counter is decreasing ($\bar{\epsilon}_{decrease}(r)$) and the energy consumption while the backoff counter is halted ($\bar{\epsilon}_{halt}(r)$) due to the busy medium. The energy consumption in both the cases could be estimated as follows:

$$\bar{\epsilon}_{backoff}(r) = \bar{\epsilon}_{decrease}(r) + \bar{\epsilon}_{halt}(r) \qquad (15)$$

where, the energy consumption $\epsilon_{decrease}$ for the fixed contention window is estimated as

$$\bar{\epsilon}_{decrease}(r) = P_{rx\_mode} \times T_{slot} \times \frac{1 + W_0}{2} \qquad (16)$$

and $\epsilon_{decrease}$ for the exponentially increased contention window is estimated as

$$\bar{\epsilon}_{decrease}(r) = P_{rx\_mode} \times T_{slot} \times \frac{1 + 2^r W_0}{2} \qquad (17)$$

where, $0 \leq$ r $\leq$ L and T_{slot} is the duration of each slot. Similarly, the energy consumption due to halting the backoff counter can be estimated as follows

$$\bar{\epsilon}_{halt}(r) = P_{rx\_mode} \times \bar{T}_{halt}(W) \times T_{data} \qquad (18)$$

Let there are N number of nodes in the network. Considering the number of ahead and non-ahead nodes with repetition of contention window size, the possible duration of halting time for the contention window size W could be estimated as:

$$\bar{T}_{halt}(W) = \sum_{i=1}^{N-1} i \cdot \left(\frac{\sum_{k=1}^{W-1} \binom{k+i-1}{i} \cdot \binom{(W-k)+(N-i-1)-1}{N-i-1}}{\binom{W+N-1}{N}} \right)$$

where, $W = 2^r W_0 - 1$ (19)

4 Performance Analysis

This section presents some numerical results and analysis of energy consumption based on the simulation results.

4.1 Simulation Setup

We consider here a low density, single-hop and fully connected WSNs without hidden terminals. All nodes in the network use the IEEE 802.15.4 CSMA as the channel access mechanism based on our extended linear feedback model and the energy consumption of packet transmissions in the network is estimated for every packet is received or sent. As per a typical WSNs, nodes are assigned time slots to wake up and to go to the power saving mode in order to maintain the power management strategies. Initially, theoretical evaluation of the total energy consumption of the network is made using our mathematical equations and taking Matlab as a tool. Then the simulation results are obtained by implementing our model with PowerTOSSIM [11]. The simulations are setup according to the IEEE 802.15.4 MAC/PHY specification and radio characteristics of IEEE 802.15.4 compliant product CC2420 [9] as listed in Table 1. The packet length is set as a constant length of 36 bytes with reference to the maximum packet length of MICAz specification[10], since MICAz is a hardware representative of the IEEE 802.15.4 platform for TinyOS and widely used today.

Table 1. Evaluation and simulation setups.

Characteristics	Value	Comments
T_s	16μs	Symbol interval
T_B	32μs	Byte interval
T_{slot}	320μs	Slot time
T_{data}	1152μs	Data trans. time
T_{cca}	640μs	CCA time
T_{ack}	544μs	ACK reception time
T_{ack_tuot}	864μs	ACK timeout
P_{rx_mode}	19.7mA	Current consumption in receive mode
P_{tx_mode}	17.4mA	Current consumption in transmit mode
V	3.3V	Typical working voltage

Finally the simulation results obtained from both Matlab and PowerTOSSIM are compared with each other to find a better energy efficient strategy by studying the binary exponential backoff (BEB) mechanism of the IEEE 802.15.4 CSMA.

4.2 Simulation Results

The simulation results obtained from both Matlab and PowerTOSSIM are described in this section. The numerical results obtained from the Matlab tool are plotted as graphs to compare the results obtained from the PowerTOSSIM simulator.

4.2.1 Numerical Analysis

The numerical results of our simulations, using the Matlab tool are presented in Fig 4.

Fig 4 presents the relationship between the expected energy consumption with the contention window size for the exponentially increased backoff mechanism of the IEEE 802.15.4 CSMA. Here, we find that energy consumption increases with the network size (N), packet generation probability (g) and contention window size. This is because the increment of N induces more number of contention windows among nodes that prolong the time of waiting for the transmission and raises the probability of collision, thereby increasing the overall energy consumption of the network. On the other hand, higher packet generation probability also leads to higher probability of collision forcing nodes to retransmit the unsuccessful packets, thereby wasting energy. Last but not the least, it is clear from Fig 4 that the large contention window causes nodes to spend more time and energy due to waiting for retransmission. As shown in Fig 5, we define the effective energy consumption means the energy consumption due to successful transmission attempts. Fig 5 shows the results of effective energy consumption for the exponentially increased contention window. From the observation, we can infer that the energy consumption can be minimized if more successful transmissions can be occurred. This could be possible by increasing the contention window size if node numbers are increased. However, higher contention window size may consume more energy, as waiting time of the nodes will be more, which is observed from Fig 5. The interesting observation from Fig 5 is that energy consumption increases with increasing the node numbers for exponentially increased contention window.

4.2.2 Analysis from PowerTOSSIM

Here, the simulation results for the energy consumption using PowerTOSSIM are presented in Fig 6 and Fig 7 for the exponentially increased contention window for different values of the traffic rate (λ). Compared to analytical results, extraordinary high energy consumption for packet transmissions is discovered in Fig 6. This is because the simulated MICAz platform stays in receive mode if it has no packet to transmit. Thus, the increment of energy consumption is greatly affected by the time duration that the simulation runs. If the simulation runs for 900 seconds, it generates traffic = 1000ms and consumes more energy than the simulation ran for 20sec which generates traffic = 100ms and consumes less energy obviously. It is to be noted that MICAz spends more energy in receiving mode than in transmission mode. Therefore, the energy consumption for traffic rate = 100ms is more than the energy consumption for the traffic rate= 10ms.

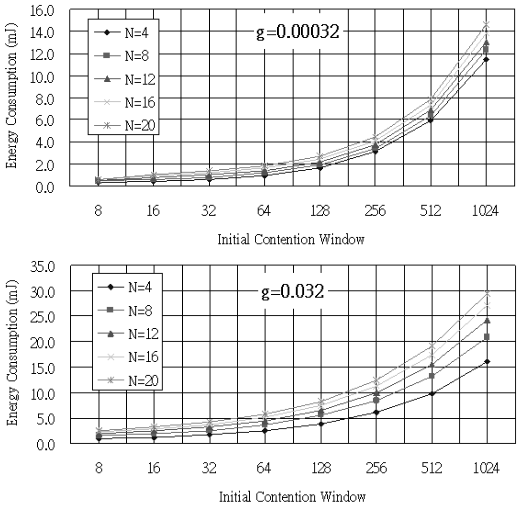

Figure 4: Expected energy consumption vs. initial contention window of BEB

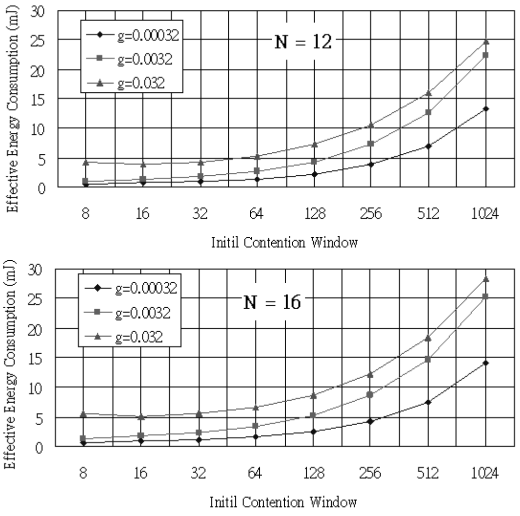

Figure 5: Effective energy consumption vs. initial contention window of BEB

The simulation results of successful delivery ratio are presented as the percentage of successful probability in Fig 7 for the exponentially increased contention window. It is important to note that from our numerical results, we find the wireless sensor nodes that use CSMA protocol with fixed contention window is more energy-efficient than the CSMA protocol with exponentially increased contention window. However, in the networks of known traffic load and population, we can find a required contention window that can achieve

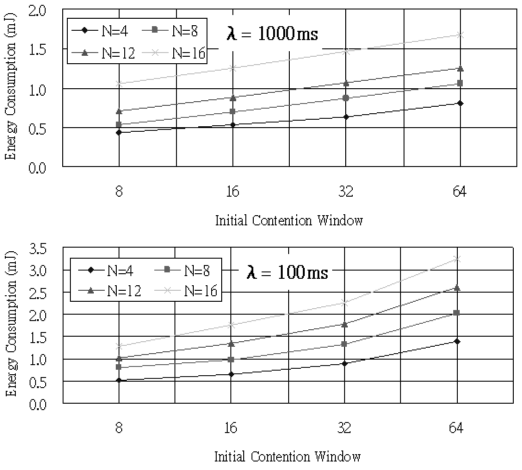

Figure 6: Energy consumption vs. initial contention window of BEB

the best effective energy consumption and could be suitable to the WSNs.

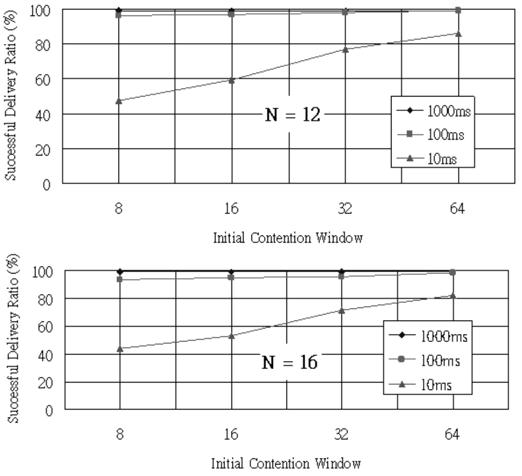

Figure 7: Successful probability vs. initial contention window of BEB

5 Conclusion

Our performance analysis show that the energy consumption of packet transmission in wireless sensor networks is increased with the increment of contention window, traffic load and network population. An optimal contention window can be derived from the use of fixed contention window to achieve the best effective energy consumption, which is defined as the minimum energy consumption for the successful packet transmission. As a result, the use of this optimal contention window, one can have a reasonable successful probability for packet transmission without extra wastage of the limited battery power.

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