Chapter 7
Localization & Positioning
Goals of this chapter

- Means for a node to determine its physical position with respect to some coordinate system (50, 27) or symbolic location (in a living room)
- Using the help of
  - Anchor nodes that know their position
  - Directly adjacent
  - Over multiple hops
- Using different means to determine distances/angles locally
Outline

- 7.1 Properties of localization and positioning procedures
- 7.2 Possible approaches
- 7.3 Mathematical basics for the lateration problem
- 7.4 Single-hop localization
- 7.5 Positioning in multihop environments
- 7.6 Positioning assisted by mobile anchors
Chapter 7.1
Properties of localization and positioning procedures
Outline

- Properties of localization and positioning procedures
- The Components of Localization Systems
Properties of localization and positioning procedures

- Physical position or logical location
  - Coordinate system: position
  - Symbolic reference: location
- Absolute coordinate: anchors are required
- Centralized or distributed computation
- Scale (indoors, outdoors, global, …)
- Limitations: GPS for example, does not work indoors
Properties of localization and positioning procedures (cont.)

Metrics

Accuracy
- how close is an estimated position to the real position?

Precision
- for repeated position determinations, how often is a given accuracy achieved?

Costs, energy consumption, …
The Components of Localization Systems

- **Distance/angle estimation**
  - Is responsible for estimating information about the distances and/or angles between two nodes

- **Position computation**
  - Is responsible for computing a node’s position based on available information concerning distances/angles and positions of reference nodes

- **Localization algorithm**
  - Is the main component of a localization system
The Components of Localization Systems (cont.)

Distance/angle estimation

- Such estimates constitute an important component of localization systems, because they are used by both the position computation and localization algorithm components.

- These methods include received signal strength indication (RSSI), time of arrival/time difference of arrival (ToA/TDoA), angle of arrival (AoA), and communication range.
The Components of Localization Systems (cont.)

*Position computation*

- When a node has enough information about distances and/or angles and positions, it can compute its own position.
- Several methods can be used to compute the position of a node.
- Such methods include trilateration, multilateration, triangulation, and the bounding box.
The Components of Localization Systems (cont.)

*Localization algorithm*

- This component determines how the information concerning distances and positions is manipulated in order to allow most or all of the nodes of a WSN to estimate their positions.

- Localization algorithms can be classified into a few categories: *distributed* or *centralized* position computation; with or without an infrastructure; relative or absolute positioning; designed for indoor or outdoor scenarios; and one-hop or multi-hop.
References


Chapter 7.2
Possible approaches
Outline

- Proximity
- Trilateration and triangulation
- Scene analysis
- Bounding box
Possible approaches

- **Proximity**
  - a node wants to determine its position or location in the proximity of an anchor

- **(Tri-/Multi-) lateration and angulation**
  - Lateration: when distances between nodes are used
  - Angulation: when angles between nodes are used

- **Scene analysis**
  - the most evident form of it is to analyze pictures taken by a camera

- **Bounding box**
  - to bound the possible positions of a node
Proximity

- Using information about a node’s neighborhood
  - Exploit finite range of wireless communication
  - E.g.: easy to determine location in a room with infrared (room number announcements)
Trilateration and triangulation

- (Tri-/Multi-) *lateration* and *angulation*
  - Using geometric properties
  - *Lateration*: distances between entities are used
  - *Angulation*: angle between nodes are used
Trilateration and triangulation (cont.)

Determining distances

- To use (multi-)lateration, estimates of distances to anchor nodes are required.

- This **ranging** process ideally leverages the facilities already present on a wireless node, in particular, the radio communication device.

- The most important characteristics are Received Signal Strength Indicator (RSSI), Time of Arrival (ToA), and Time Difference of Arrival (TDoA).
Distance estimation

**RSSI (Received Signal Strength Indicator)**

- Send out signal of known strength, use received signal strength and path loss coefficient to estimate distance

\[ P_{\text{rcvd}} = c \frac{P_{\text{tx}}}{d^\alpha} \Leftrightarrow d = \sqrt[\alpha]{\frac{c P_{\text{tx}}}{P_{\text{rcvd}}}} \]
Distance estimation

*RSSI (cont.)*

- **Problem: Highly error-prone process:**
  - Caused by fast fading, mobility of the environment
  - Solution: repeated measurement and filtering out incorrect values by statistical techniques
Distance estimation

**RSSI (cont.)**

- **Problem: Highly error-prone process:**
  - Cheap radio transceivers are often not calibrated
    - Same signal strength result in different RSSI
    - Actual transmission power different from the intended power
    - Combination with multipath fading
    - Signal attenuation along an indirect path is higher than along a direct path
  - **Solution: No!**
Distance estimation

**RSSI (cont.)**

PDF of distances in a given RSSI value
Distance estimation

*ToA (Time of arrival)*

- **Use**
  - time of transmission,
  - propagation speed

- **Problem: Exact time synchronization**
  - Usually, sound wave is used
  - But propagation speed of sound depends on temperature or humidity
Distance estimation

*TDoA (Time Difference of Arrival)*

- Use two different signals with different propagation speeds
  - Compute difference between arrival times to compute distance
  - Example: ultrasound and radio signal *(Cricket System)*
    - Propagation time of radio negligible compared to ultrasound
- Problem: expensive/energy-intensive hardware
Determining angles

- **Directional antennas**
  - Node mount a directional antennas
    - Supporting infrastructure anchors
  - Multiple antennas mounted on a device at known separation
    - Measuring the time difference between a signal’s arrival at the different antennas
Trilateration and triangulation (cont.)

*Triangulation*

- The unknown node estimates its angle to each of the three reference nodes and, based on these angles and the positions of the reference nodes (which form a triangle)
- Computes its own position using simple trigonometrical relationships
Scene analysis

- Analyze characteristic properties of the position of a nods in comparison with premeasured properties
  - Radio environment has characteristic “fingerprints”
Bounding Box

- The bounding box method proposed in uses squares instead of circles as in tri-lateration to bound the possible positions of a node.

- For each reference node $i$, a bounding box is defined as a square with its center at the position of this node $(x_i, y_i)$, with sides of size $2d_i$ (where $d$ is the estimated distance) and with coordinates $(x_i-d_i, y_i-d_i)$ and $(x_i+d_i, y_i+d_i)$. 
Bounding Box (cont.)

- Using range to anchors to determine a bounding box
- Use center of box as position estimate
References


Chapter 7.3
Mathematical basics for the lateration problem
Outline

- Solution with three anchors and correct distance values
- Solving with distance errors
Solution with three anchors and correct distance values

- Assuming distances to three points with known location are exactly given
- Solve system of equations (Pythagoras!)

- \((x_i, y_i)\) : coordinates of anchor point \(i\),
- \(r_i\) distance to anchor \(i\)
- \((x_u, y_u)\) : unknown coordinates of node

\[
(x_i - x_u)^2 + (y_i - y_u)^2 = r_i^2 \text{ for } i = 1, \ldots, 3
\]
Solution with three anchors and correct distance values (cont.)

\[(x_i - x_u)^2 + (y_i - y_u)^2 = r_i^2 \text{ for } i = 1, \ldots, 3\]

\[
\begin{align*}
(x_1 - x_u)^2 - (x_3 - x_u)^2 + (y_1 - y_u)^2 - (y_3 - y_u)^2 &= r_1^2 - r_3^2 \\
(x_2 - x_u)^2 - (x_u - x_u)^2 + (y_2 - y_u)^2 - (y_3 - y_u)^2 &= r_2^2 - r_3^2
\end{align*}
\]

\[\therefore = 3\]
Trilateration as matrix equation

- We get
- Rewriting as a matrix equation:

\[ 2(x_3 - x_1)x_u + 2(y_3 - y_1)y_u = (r_1^2 - r_3^2) - (x_1^2 - x_3^2) - (y_1^2 - y_3^2) \]
\[ 2(x_3 - x_2)x_u + 2(y_3 - y_2)y_u = (r_2^2 - r_3^2) - (x_2^2 - x_3^2) - (y_2^2 - y_3^2) \]

\[
2 \begin{bmatrix}
  x_3 - x_1 & y_3 - y_1 \\
  x_3 - x_2 & y_3 - y_2 
\end{bmatrix}
\begin{bmatrix}
  x_u \\
  y_u 
\end{bmatrix} =
\begin{bmatrix}
  (r_1^2 - r_3^2) - (x_1^2 - x_3^2) - (y_1^2 - y_3^2) \\
  (r_2^2 - r_3^2) - (x_2^2 - x_3^2) - (y_2^2 - y_3^2) 
\end{bmatrix}
\]

\[ = 3 \]
Solving with distance errors

- What if only distance estimation $r_i^0 = r_i + \varepsilon_i$ available?
- Use multiple anchors, overdetermined system of equations

\[
2 \begin{bmatrix}
  x_n - x_1 & y_n - y_1 \\
  \vdots & \vdots \\
  x_n - x_{n-1} & y_n - y_{n-1}
\end{bmatrix}
\begin{bmatrix}
x_u \\
y_u
\end{bmatrix} = \\
\begin{bmatrix}
  (r_1^2 - r_n^2) - (x_1^2 - x_n^2) - (y_1^2 - y_n^2) \\
  \vdots \\
  (r_{n\,1}^2 - r_n^2) - (x_{n\,1}^2 - x_n^2) - (y_{n\,1}^2 - y_n^2)
\end{bmatrix}
\]

- Use $(x_u, y_u)$ that minimize mean square error,
  - i.e, $\|Ax - b\|_2$
Minimize mean square error

- Look at square of the Euclidean norm expression
  - (note that $\|v\|_2^2 = v^Tv$ for all vectors $v$)

$$\|Ax - b\|_2^2 = (Ax - b)^T(Ax - b) = x^TA^TAx - 2x^TA^Tb + b^Tb$$

$$2A^TAx - 2A^Tb = 0 \iff A^TAx = A^Tb$$

- Look at derivative with respect to $x$, set it equal to 0
Chapter 7.4
Single-hop localization
Outline

- Active Badge
- Active office
- RADAR
- Cricket
- Overlapping connectivity
- Using angle of arrival information
Active Badge

- Uses diffused infrared as transmission medium
- Exploits the natural limitation of infrared waves by walls as a delimiter for its location granularity
- A badge periodically sends a globally unique identifier via infrared to receivers, at least one of which is installed in every room
Active office

- Use ultrasound
- With receivers placed at well-known position, mounted in array at the ceiling of a room
- Devices for which the position is to be determined act as ultrasound senders
Active office (cont.)

- **Process:**
  - Central controller sends a radio containing the devices ‘s address
  - The devices upon receiving this radio message, sends out a short ultrasound pulse
  - The receiver array compute the difference of the arrival time of the radio and ultrasound pulse
RADAR

- Scene analysis techniques
- Both the anchors and the mobile device can be used to send the signal, which is then measured by the counterpart device(s)
- Uses RF signal strength (SS) from multiple receiver locations to triangulate the user’s coordinates
- Can be used for location aware applications
Cricket

- Combines radio wave and ultrasound pulses to allow measuring of the TDoA

**Objectives:**
- User privacy
- Decentralized administration
- Low cost
- Granularity
Overlapping connectivity

- Without any numeric range measurement
- Use connectivity to a set of anchors
- The underlying assumption is that transmissions from an anchor can be received within a circular area of known radius
Overlapping connectivity (cont.)

Positioning using connectivity information to multiple anchors
Overlapping connectivity (cont.)

- **Process:**
  - Anchor nodes periodically send out transmissions identifying themselves
  - A node has received these announcements, it can determine that it is in the intersection of the circles
  - Suppose node knows about all the anchors
    - Anchor announcements are not received implies that the node is outside the respective circles

- **Problem:**
  - accuracy depends on the number of anchors
Using angle of arrival information

- Idea: Use antenna array to measure direction of neighbors
- Special landmarks have compass + GPS, broadcast location and bearing
- Flood beacons, update bearing along the way
- Once bearing of three landmarks is known, calculate position
References

Chapter 7.5
Positioning in multihop environments
Outline

- Connectivity in a multihop network
- Multihop range estimation
- Iterative and collaborative multilateration
- Probabilistic positioning description and propagation
Connectivity in a multihop network

- Assume that the positions of $n$ anchors are known and the positions of $m$ nodes is to be determined, that connectivity between any two nodes is only possible if nodes are at most $R$ distance units apart, and that the connectivity between any two nodes is also known.

- The fact that two nodes are connected introduces a constraint to the feasibility problem – for two connected nodes, it is impossible to choose positions that would place them further than $R$ away.
Multihop range estimation

- How to estimate range to a node to which no direct radio communication exists?
  - No RSSI, TDoA, …
  - But: Multi-hop communication is possible
Multihop range estimation (cont.)

- Idea 1: Count number of hops, assume length of one hop is known (*DV-Hop*)
  - Start by counting hops between anchors, divide known distance

- Idea 2:
  - If range estimates between neighbors exist, use them to improve total length of route estimation in previous method (*DV-Distance*)
Multihop range estimation (cont.)

*DV-Based Scheme*

- Must work in a network which is dense enough DV-hop approach used the hop of the shortest path to approximately estimate the distance between a pair of nodes
- Drawback: Requires lots of communications
Discussion

- **Number of anchors**
  - Euclidean method increase accuracy as the number of anchors goes up
  - The “distance vector”-like methods are better suited for a low ratio for anchors

- **Uniformly distributed network**
  - Distance vector methods perform less well in anisotropic networks
  - Euclidean method is not very sensitive to this effect
Iterative multilateration

I:

II:

III:

IV:
Iterative multilateration

- Assume some nodes can hear at least three anchors (to perform triangulation), but not all

Idea:
- let more and more nodes compute position estimates,
- spread position knowledge in the network

Problem:
- Errors accumulate
- When not all nodes in the network will have three nodes with position estimates
Collaborative multilateration

- **Defining participating nodes**
  - nodes that have at least three anchors or other participating nodes as neighbors, making nodes A and B participating nodes

- For such participating nodes, positioning can be solved
Collaborative multilateration

- Needs at least three *independent* references to anchor nodes

- Such nodes are called *sound*
  - nodes A, B, and C are all sound

- Soundness can be detected during the initial position estimation
Iterative multilateration

- **Solution 1: Participating nodes**
  - Have at least three anchors or other participating nodes as neighbors

- **Solution 2: Sound**
  - Have at least three independent references to anchor nodes
  - The path to the anchors have to be edge-disjoint.

- **Discussion:**
  - Solution 2 is suited to low anchor ratios
Probabilistic position description

- Similar idea to previous one, but accept problem that position of nodes is only probabilistically known
  - Represent this probability explicitly, use it to compute probabilities for further nodes
Probabilistic position description

(a) Probability density function of a node positions after receiving a distance estimate from one anchor

(b) Probability density functions of two distance measurements from two independent anchors

(c) Probability density function of a node after intersecting two anchor’s distance measurements
References


Chapter 7.6
Positioning assisted by mobile anchors
Outline

- Localization with a Mobile Beacon
- Mobile-assisted Localization
- APIT
- MCL
- MSL
- DRLS
- IMCL
Localization with a Mobile Beacon

- Some recent work has proposed the use of mobile beacons to assist the nodes of a WSN in estimating their positions.

- The mobile beacon travels through the sensor field broadcasting messages that contain its current coordinates.

- When a free node receives more than three messages from the mobile beacon, it computes its position, using a probabilistic approach, based on the received coordinates and RSSI distance estimations.
Localization with a Mobile Beacon (cont.)

Mobile beacon trajectory
Localization with a Mobile Beacon (cont.)

- Corresponding to the RSSI measurement and the position of the beacon \((x_B, y_B)\) (included in the beacon packet), each node receiving the beacon constructs a constraint on its position estimate:

\[
Constraint(x, y) = PDF_{RSSI}(d((x, y), (x_B, y_B)))
\]

\[
\forall (x, y) \in [(x_{min}, x_{max}) \times (y_{min}, y_{max})]
\]
Localization with a Mobile Beacon (cont.)

Once the constraint is computed, each node applies Bayesian inference to compute its new position estimate $NewPosEst$ from its old position estimate $OldPosEst$ and the new constraint $Cons$:

$$NewPosEst(x, y) = \frac{\int_{x_{min}}^{x_{max}} \int_{y_{min}}^{y_{max}} OldPosEst(x, y) \times Cons(x, y)}{\int_{x_{min}}^{x_{max}} \int_{y_{min}}^{y_{max}} OldPosEst(x, y) \times Cons(x, y)} \forall (x, y) \in [(x_{min}, x_{max}) \times (y_{min}, y_{max})].$$
Localization with a Mobile Beacon (cont.)

- The initial position estimate is initialized to a constant value, as in the beginning, all positions in the deployment area are equally likely.

- The beacon sends beacon packets at each of the positions marked on the trajectory.
Localization with a Mobile Beacon (cont.)
Reference

Mobile-assisted Localization (MAL)

- Obstructions, especially in indoor environments
- Sparse node deployments
- Geometric dilution of precision (GDOP)
- Hence, finding 4 reference points for each node for localization is difficult
MAL (cont.)

- Find four stationary nodes
- Using Specific MAL Movement Strategy To Construct A Rigid Graph And Compute Inter-node Distance
- Using Anchor-Free Localization (AFL) to Compute Coordinates and Optimize Solution
MAL (cont.)

*Movement Strategy*

- Calculating distances among 4 (or more) nodes
  To compute the pairwise distances between $j \geq 4$ nodes $n_1, n_2, \ldots, n_j$

- We require at least $\left\lceil \frac{3j-5}{j-3} \right\rceil$ mobile positions (to reduce the degree of freedom to 0)

  When $j=4$, the $\left\lceil \frac{3j-5}{j-3} \right\rceil=7$
MAL (cont.)

Movement Strategy

Initialize:

a. Find Four Stationary Nodes that are visible (distance are measurable) to mobile location
MAL (cont.)

*Movement Strategy*

**Initialize:**

b. Move the mobile to at least seven nearby locations and measure distances.
MAL (cont.)

Movement Strategy

Initialize:

c. Compute the pairwise distances between the four stationary nodes
MAL (cont.)

Movement Strategy

Initialize:

    d. Localize the resulting tetrahedron using Rigidity Theorem
MAL (cont.)

Movement Strategy

Loop:

a. Pick a stationary node that has been localized but has not yet examined by this loop

b. Move the mobile around the stationary node and search for non-localized stationary node and 0, 1 or 2 additional localized nodes

c. For each such mobile position: Compute the distance between those 2, 3, or 4 stationary nodes and localize the node if it has 4 know distances.

d. Terminates the loop if every stationary node has been localized or no more progress can be made.
Reference

APIT (Approximate point in triangle)

- By pure connectivity information
- Idea: decide whether a node is within or outside of a triangle formed by any three anchors
- However, moving a sender node to determine its position is hardly practical!
- Solution:
  - inquire all its neighbors about their distance to the given three corner anchors
APIT (cont.)

- **Inside a triangle**
  - irrespective of the direction of the movement, the node must be closed to at least one of the corners of the triangle
Outside a triangle:

- There is at least one direction for which the node’s distance to all corners increases.
APIT (cont.)

- Approximation: Normal nodes test only directions towards neighbors

A. Inside Case

B. OutSide Case
APIT (cont.)

Grid-Based Aggregation

- Narrow down the area where the normal node can potentially reside

- anchor node
- normal node
Reference

MCL (Monte-Carlo Localization)

- **Assumptions**
  - Time is divided into several time slots
  - Moving distance in each time slot is randomly chosen from $[0, V_{\text{max}}]$
  - Each anchor node periodically forwards its location to two-hop neighbors

- **Notation**
  - $R$ - communication range
MCL (cont.)

- Each normal node maintains 50 samples in each time slot
  - Samples represent the possible locations
  - The sample selection is based on previous samples
  - Sample \((x, y)\) must satisfy some constraints
    - Located in the anchor constraints
MCL (cont.)

- **Anchor constraints**
  - Near anchor constraint
    - The communication region of one-hop anchor node (near anchor)
  - Farther anchor constraint
    - The region within \((R, 2R]\) centered on two-hop anchor (farther anchor)
MCL (cont.)

Environment

- Anchor node
- Normal node

Diagram showing various nodes labeled as $A_1$, $A_2$, $A_3$, and $A_4$.
MCL (cont.)

Initial Phase

- Anchor node
- Normal node
- Sample in the last time slot

Diagram showing nodes and markers.
MCL (cont.)

Prediction Phase & Filtering Phase

- Sample in this time slot
- Sample in the last time slot
- Anchor node
- Normal node
MCL (cont.)

**Prediction Phase & Filtering Phase**

- Anchor node
- Normal node
- Sample in the last time slot
- Sample in this time slot

2010/5/24
MCL (cont.)

*Estimative Location*

- the average of samples

Sample in this time slot
- Estimative position
- Anchor node
- Normal node
MCL (cont.)

Repeated Prediction Phase & Filter Phase

In the next time slot

- **Sample in this time slot**
- **Sample in the last time slot**
- **Anchor node**
- **Normal node**
Reference

MSL

Mobile and Static sensor network Localization

- In MSL, we assign a weight to each node. Every node uses the weights of its neighbors (rather than weights of samples of neighbors) to weight its samples.

- After the weights are computed, MSL computes a single location estimate (the weighted mean of samples) and a closeness value.

- Each node broadcasts to its neighbors this estimate and its closeness value (but not its samples). Thus, the communication cost drops significantly.
MSL (cont.)

Differences with MCL

- Improve on MCL and generalize it in several ways.
  - First, modify the sampling procedure in MCL to allow our algorithm to work in static networks.
  - Second, each node uses information from only those neighbors that have better location estimates (measured using the \textit{closeness} parameter) than it.
  - Third, modify the sampling procedure and allow samples to have weights greater than a threshold value, $\beta$. 

Reference

DRLS

* Distributed Range-Free Localization Scheme

- There are three phases in the DRLS algorithm.
  - Phase 1 – Beacon exchange
  - Phase 2 – Using improved grid-scan algorithm to get initial estimative location
  - Phase 3 – Refinement
DRLS (cont.)

Beacon Exchange

- Beacon exchange via two-hop flooding
DRLS (cont.)

Improved Grid-Scan Algorithm

- Calculate the overlapping rectangle
DRLS (cont.)

*Improved Grid-Scan Algorithm*

- Divide the ER into small grids
  - The initial value of the grid is 0

![Diagram showing grid division and nodes](attachment:image.png)
DRLS (cont.)

*Improved Grid-Scan Algorithm*

- Initial estimative location
  - Apply centroid formula to grids with the maximum grid value
DRLS (cont.)

Refinement

- Repulsive virtual force (VF)
  - Induced by farther anchor nodes

- $D_{\text{invasion}}$: the maximum distance that the farther anchor invades the estimative region along the direction from the farther anchor towards the initial estimative location
DRLS (cont.)

Refinement

- $VF_i$: virtual force induced by farther anchor $i$
- $D_{\text{invasion}}_i$: the maximum distance that the farther anchor $i$ invades the estimative region along the direction from $i$ towards the initial estimative location
- $V_{i,j}$: the unit vector in the direction from the farther anchor $i$ towards the initial estimative location $j$
- $VF_i = V_{i,j} \cdot D_{\text{invasion}}_i$
DRLS (cont.)

Refinement

- Resultant Virtual Force (RVF)
- $\text{RVF} = \Sigma \text{VF}_i$
DRLS (cont.)

Refinement

- $D_i$: the moving distance caused by the farther anchor $i$

- $D_{imax}$: the maximum moving distance caused by the farther anchor $i$

\[
\frac{D_i}{D_{imax}} = \frac{D_{invasion_i}}{D_{invasion_{imax}}}
\]
DRLS (cont.)

Refinement

- $D_{move_i}$: the moving vector caused by the farther anchor $i$
- $V_{i,j}$: the unit vector in the direction from the farther anchor $i$ towards the initial estimative location $j$
- $D_{move_i} = V_{i,j} \cdot D_i$
DRLS (cont.)

Refinement

- $D_{move}$: the final moving vector

$$D_{move} = \Sigma D_{move_i}$$
Reference

IMCL

Improve MCL Localization Scheme

- **Improvements**
  - Dynamic number of samples
    - According to the overlapping region of anchor constraints
  - Restricted samples
    - Anchor constraints
    - The estimative locations of neighboring normal nodes
  - Predicted moving direction of the normal node
    - Be used to increase the localization accuracy
IMCL (cont.)

- Phase 1- Sample Selection Phase
- Phase 2- Neighbor Constraints Exchange Phase
- Phase 3- Refinement Phase
Dynamic sample number

- Sampling Region
  - The overlapping region of anchor constraints
- Difficult to calculate
- Estimative Region (ER)
  - A rectangle surrounding the sampling region
The number of samples ($k$)

\[
k \leq \text{Max\_Num}
\]

\[
k = \left\lfloor \text{Max\_Num} \times \frac{\text{ER}_\text{Area}}{\text{ER}_\text{Threshold}} \right\rfloor
\]

- $\text{ER}_\text{Area}$ — the area of ER
- $\text{ER}_\text{Threshold}$ — the threshold value
- $\text{Max\_Num}$ — the upper bound of sample number

In our simulations, $\text{ER}_\text{Threshold} = 4R^2$
IMCL (cont.)

Sample Selection Phase

- Using the prediction and filtering phase of MCL
  - Samples are randomly selected from the region extended $V_{max}$ from previous samples
  - Filter new samples
    - Near anchor constraints
    - Farther anchor constraints
An additional constraint

- Samples must locate on the communication region of neighboring normal nodes
- The localization error may increase

Send the **possible location region** to neighbors instead of the estimative position
IMCL (cont.)

Neighbor Constraints Exchange Phase

- The possible location region
  - The distribution of samples are selected in phase I

Step 1: Sensor A constructs a coordinate axis and uses \((C_x, C_y)\) as origin

Step 2: The coordinate axis is separated into eight directions

Central position in phase 1
Step 3: The samples are also divided into eight groups according to the angle $\theta$ with $(C_x, C_y)$

$$\theta = \tan^{-1}\left(\frac{S_y - C_y}{S_x - C_x}\right)$$

Sample in the this time slot

Central position in phase 1
IMCL (cont.)

Neighbor Constraints Exchange Phase

Step 4: Using the longest distance within group as radius to perform sector

the possible location region described by eight sectors and \((C_x, C_y)\)

Sample in the this time slot

Central position in phase 1
IMCL (cont.)

Neighbor Constraints Exchange Phase

- Neighbor constraint
  - Extend R from the possible located region

Each sensor broadcasts its neighbor constraint region once
IMCL (cont.)

Refinement Phase

- **Samples are filtered**
  - Neighbor constraints
    - Receive from neighboring normal nodes
  - Moving constraint
    - Predict the possible moving direction

- **When sample is not satisfy the constraints**
  - Normal node generates a valid sample to replace it
IMCL (cont.)

Reﬁnement Phase

- Neighbor constraints
  - Sample $S_1$ is a valid sample
    - Satisﬁed both neighbor constraints of $N_2$ and $N_3$
  - Sample $S_2$ is an invalid sample
    - Only satisﬁed the neighbor constraint of $N_3$
IMCL (cont.)

Refinement Phase

Moving constraint

- The prediction of nodes moving direction is $[\theta \pm \Delta \Phi]$
If prediction is right, sample must be located in $\{\theta \pm \Delta \Phi\}$ from $E_{t-2}$

Sample 1 satisfies moving constraint

Sample 2 does not satisfy moving constraint
Normal node calculates the estimative position \( E_t (E_x, E_y) \) of samples

- \( E_x = \frac{\sum_{i=1}^{k} x \text{ coordinate of sample } i}{k}, \quad k = \text{sample number} \)
- \( E_y = \frac{\sum_{i=1}^{k} y \text{ coordinate of sample } i}{k}, \quad k = \text{sample number} \)
Reference

Conclusions

- Determining location or position is a vitally important function in WSN, but fraught with many errors and shortcomings
  - Range estimates often not sufficiently accurate
  - Many anchors are needed for acceptable results
  - Anchors might need external position sources (GPS)
  - Multilateration problematic (convergence, accuracy)