Algo hw 16 solution

Problem :

Give an efficient algorithm to count the total number of paths in a directed acyclic graph. Analyze your algorithm

24.2 – 4

We count the number of directed paths in a directed acyclic graph $G = (V, E)$ as follows. First perform a topological sort of the input. Then for all $v \in V$ compute, $B(v)$ defined as follows.

$$B(v) = \begin{cases} 1 & v \text{ is last in the order} \\ 1 + \sum_{(v,w)\in E} B(w) & \text{otherwise} \end{cases}$$

$B(v)$ computes the number of directed paths beginning at v since if v is last in the order the only path starting at v is the empty one. Otherwise for each node w, $(v, w) \in E$, (v, w) concatenated with the paths from w and the empty path are the paths starting from v. We then compute the number of directed paths after v in the topological order. We denote this by $D(v)$ and we obtain the following.

$$D(v) = B(v) + \sum_{(v,w)\in E} D(w)$$

Since the nodes of G are ordered topologically $B(v)$ and $D(v)$ can be computed in linear. Thus the total running time is $O(E + V)$.

24.3-5

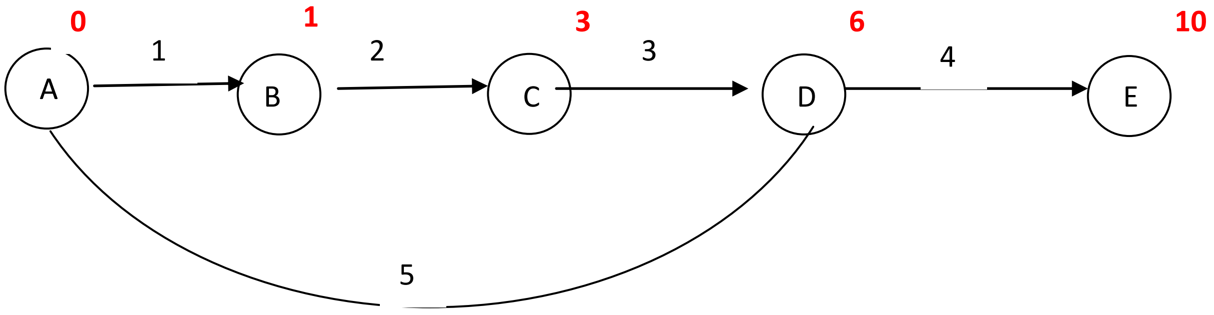
Problem :

Professor Newman thinks that he has worked out a simpler proof of correctness for Dijkstra's algorithm. He claims that Dijkstra's algorithm relaxes the edges of every shortest path in the graph in the order in which they appear on the path, and therefore the path-relaxation property applies to every vertex reachable from the source. Show that the professor is mistaken by constructing a directed graph for which Dijstra's algorithm could relax the edges of a shortest path out of order.

Sol :

按題意，此演算法對每個最小路徑中依序出現的 edge 做 realax，這是錯的。下圖為反例。

照題目的演算法 relax 的順序為 e(A,B) -> e(B,C) -> e(C,D) -> e(D,E) -> e(A,D)，A 到 E 的長度為 10



但是實際上，A 到 E 的路徑長度應該是 9。所以 professor Newman 的方法是錯的。